<u>SYSTEMS OF UNITS (</u>SI .)

there are six principal units from which the units of all other physical quantities can be derived. Table 1.1 shows the six units, their symbols, and the physical quantities they represent.

TABLE 1.1 The six basic SI units.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd

One great advantage of the SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit. Table 1.2 shows the SI prefixes and their symbols.
For example, the following are expressions of the same distance in meters (m):
600,000,000 mm 600,000 m 600 km

TABLE 1.2 The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

<u>voltage and current</u>

Voltage and Current are the two basic variables in electric circuits.

The concept of electric charge is the underlying principle for explaining all electrical phenomena. Also, the most basic quantity in an electric circuit is the *electric charge.*

> Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

The following points should be noted about electric charge:

1. The coulomb is a large unit for charges. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or μC.[1]
2. According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge $e = -1.602 \times 10^{-19}$ C.
3. The *law of conservation of charge* states that charge can neither be created nor destroyed, only transferred. Thus the algebraic sum of the electric charges in a system does not change.

When a conducting wire (consisting of several atoms) is connected to a battery (a source of electromotive force), the charges are compelled to move; positive charges move in one direction while negative charges move in the opposite direction. This motion of charges creates electric current. It is conventional to take the current flow as the movement of positive charges, that is, opposite to the flow of negative charges, as Fig. 1.3 illustrates.

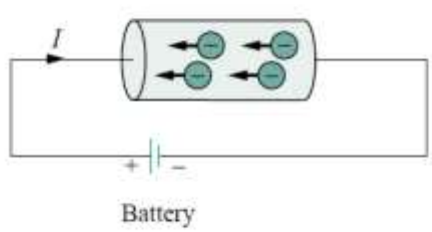


Fig.(1.3) Electric current due to flow of electronic charge in a conductor

Mathematically, the relationship between current *i*, charge *q*, and time *t* is:

$$i = \frac{dq}{dt}$$

------------------------------(1-1)

where current is measured in amperes (A), and
1 ampere = 1 coulomb/second

The charge transferred between time *t*0 and *t* is obtained by integrating both sides of Eq. (1.1). We obtain

$$q = \int_{t_0}^{t} i\,dt$$

A direct current (dc) is a current that remains constant with time.

fig.(1.4)

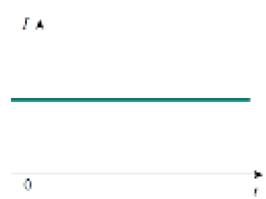
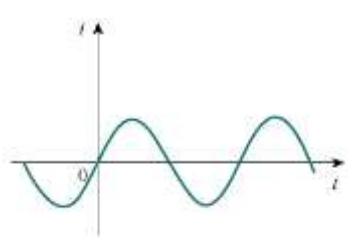


Fig.(1.4)

An alternating current (ac) is a current that varies sinusoidally with time.

fig.(1.5)



Fig(1.5)

Ex 1: The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate: the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

Ex2: Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$q = \int_{t=1}^{2} i\, dt = \int_{1}^{2} (3t^2 - t)\, dt$$

$$= \left(t^3 - \frac{t^2}{2}\right)\Bigg|_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5 \text{ C}$$

Voltage

to move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. 1.3. This emf is also known as *voltage* or *potential difference*.

The voltage *vab* between two points *a* and *b* in an electric circuit is the energy (or work) needed to move a unit charge from *a* to *b*; mathematically

$$v_{ab} = \frac{dw}{dq}$$

where *w* is energy in joules (J) and *q* is charge in coulombs (C). The voltage *vab* or simply *v* is measured in volts (V)

1 volt = 1 joule/coulomb = 1 newton meter/coulomb

Then:

> Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).

Figure 1.6 shows the voltage across an element (represented by a rectangular block) connected to points *a* and *b*. The plus *(+)* and minus *(−)* signs are used to define reference direction or voltage polarity. The *vab* can be interpreted in two ways: (1) point *a* is at a potential of *vab* volts higher than point *b*, or (2) the potential at point *a* with respect to point *b* is *vab*.

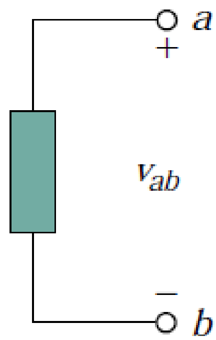


Fig.(1.6)
Polarity of voltage *vab*

It follows logically that in general:

$$vab = -\,vba$$

For example, in Fig. 1.7, we have two representations of the same voltage. In Fig. 1.7(a), point *a* is+9V above point *b*; in Fig. 1.7(b), point *b* is −9 V above point *a*. We may say that in Fig. 1.7(a), there is a 9-V *voltage drop* from *a* to *b* or equivalently a 9-V *voltage rise* from *b* to *a*. In other words, a voltage drop from *a* to *b* is equivalent to a voltage rise from *b* to *a*.

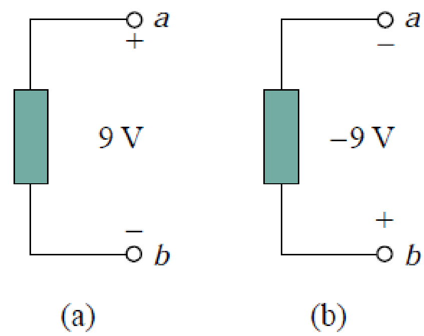
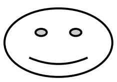


Fig.(1.7)
Two equivalent representations of the same voltage *vab*
(a) point *a* is 9 V above point *b*
(b) point *b* is −9 V above point *a*.



Keep in mind that electric current is always *through* an element and that electric voltage is always *across* the element or between two points.

<u>circuit elements</u>

Circuit elements are classified into

<u>Passive elements</u>

1-Resistor

2- Inductor

3- Capacitor

<u>Active elements such as</u>

1-Voltage source like batteries

2-Current source

3-Generators

4-Operational amplifier

There are two types of elements found in electric circuits: *passive* elements and *active* elements. An active element is capable of generating energy while a passive element is not.

<u>voltage and current sources</u>

The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources:

a. <u>Independent sources</u>
b. <u>dependent sources</u>

> An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit variables.

<u>independent voltage source</u> delivers to the circuit whatever current is necessary to maintain its terminal voltage batteries and generators may be regarded as approximations to ideal voltage sources. Figure 1-8 shows the symbols for independent voltage sources.

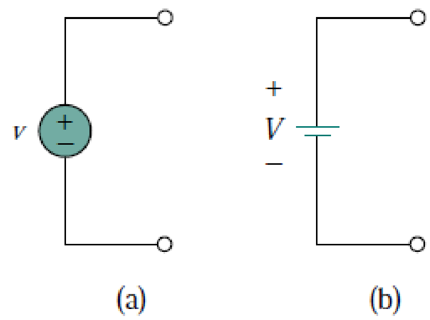


Fig1-8
Symbols for independent voltage sources

an ideal independent current source is an active element that provides a specified current completely independent of the voltage across the source That is,
the current source delivers to the circuit whatever voltage is necessary to maintain the designated current. The symbol for an independent current source is displayed in Fig. 1-9 , where the arrow indicates the direction of current *i*

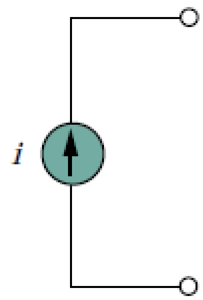


Fig. 1-9
Symbol for independent current source.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. (1-10)

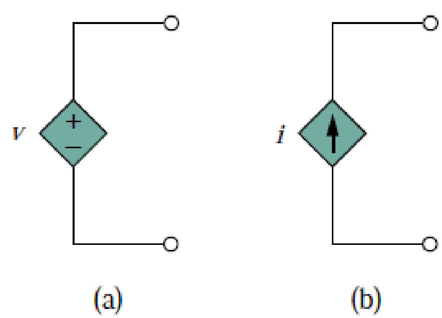


Fig. (1-10)
Symbols for:
(a) Dependent voltage source,
(b) Dependent current source

<u>Ideal &actual source</u>

1- The term (ideal voltage source) means that the internal resistance (r_s) of the source (voltage source) equal zero.

2- The term (ideal current source) means that the internal resistance (r_s) of the source (current source) equal ∞ fig.(1-11)

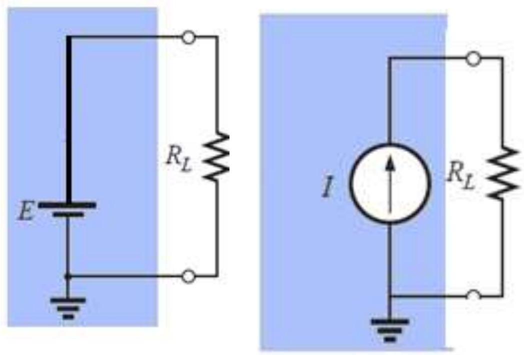


Fig (1-11)

Ideal sources

3- The term (actual source) means that there is an internal resistance (r_s) of the source (voltage source or current source).fig.(1-12)

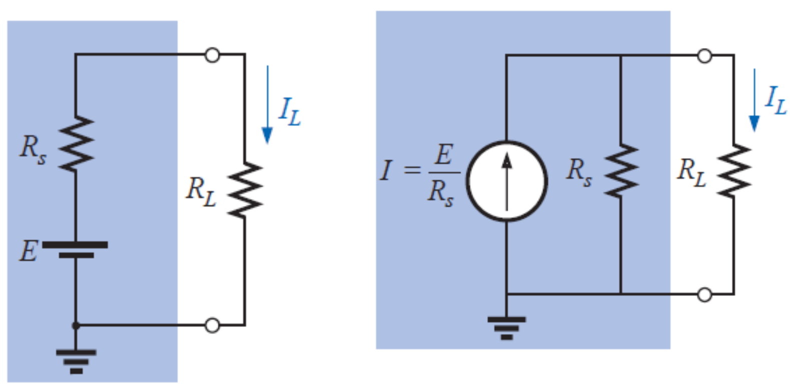


Fig (1-12)

Actual sources

Voltage and current source conversion

source can be converted to the other type. Fig (1-13).

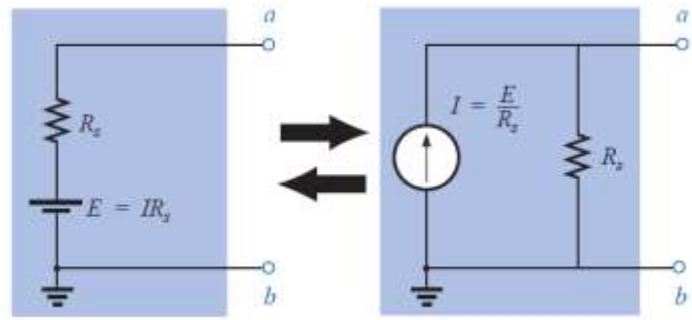


Fig (1-13)

Source conversion.

Voltage source to current source and vice versa ← العكس بالعكس

electrical resistance and conductance

Passive elements

Resistance of the material:

The flow of charge through any material encounters an opposing force due to the collisions between electrons and between electrons and other atoms in the material, *which converts electrical energy into another form of energy such as heat,* is called the resistance of the material . The unit of measurement of resistance is the ohm, for which the symbol is (**Ω**)
The circuit symbol for resistance appears in Fig. (1.1)

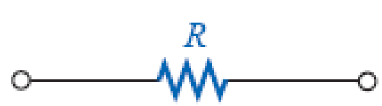


Fig. (1-14)

Resistance symbol

The resistance of any material with a uniform cross-sectional area is determined by the following four factors:

1. Material resistivity
2. Length
3. Cross-sectional area
4. Temperature

Conductors will have low resistance levels, while insulators will have high resistance characteristics.
At a fixed temperature of 20°C (room temperature), the resistance is related to the other three factors by:

$$R = \rho \frac{l}{A} \quad \text{(ohms, } \Omega\text{)}$$ ____________(1.1)

ρ: ohm-centimeters
l: centimeters
A: square centimeters

Where ρ (Greek letter rho) is a characteristic of the material called the resistivity, l is the length of the sample, and A is the cross-sectional area of the sample.

CONDUCTANCE (G)

By finding the reciprocal of the resistance of a material, we have a measure of how well the material will conduct electricity. The quantity is called conductance, has the symbol *G*, and is measured in *Siemens* (S).

$$G = \frac{1}{R} \quad \text{(siemens, S)}$$ ----------------- (1.2)

In equation form, the conductance is determined by:

$$G = \frac{A}{\rho l} \quad \text{(S)}$$ ------------------ (1.3)

Ohm's law

$$R = \frac{E}{I} \quad \text{(ohms, } \Omega\text{)}$$

Or

$$I = \frac{E}{R} \quad \text{(amperes, A)}$$

Or

$$E = IR \quad \text{(volts, V)}$$

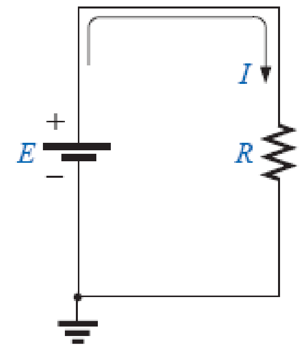


Fig (1-15)

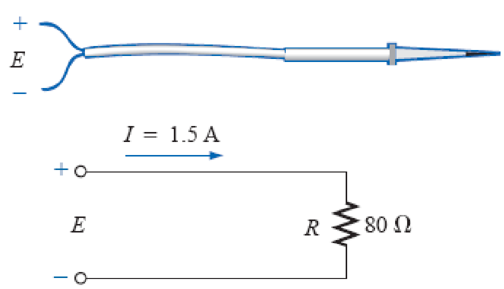
Basic circuit

Ex.3 Determine the current resulting from the application of a 9V battery across a network with a resistance of 2.2 Ω.

Sol.

$$I = \frac{E}{R} = \frac{9\text{ V}}{2.2\ \Omega} = \mathbf{4.09\ A}$$

Ex. 4 Calculate the voltage that must be applied across the soldering iron of Fig.(1-16) to establish a current of 1.5 A through the iron if its internal resistance is 80 Ω.



Fig(1-16)

Sol.

$$E = IR = (1.5\text{ A})(80\ \Omega) = \mathbf{120\ V}$$

<u>Voltage and current source conversion</u>

sources can be converted to the other type. Fig (1-17).

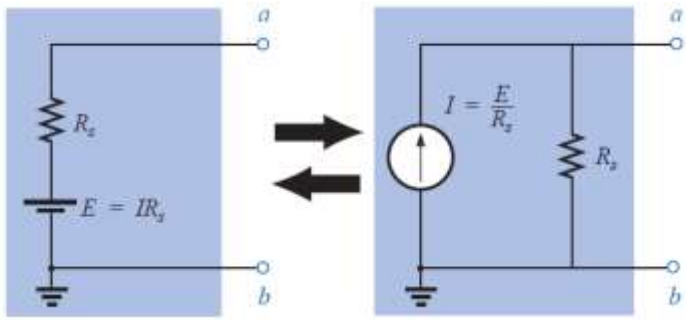


Fig (1-17)

Source conversion.

Voltage source to current source and <u>vice versa</u> ← العكس بالعكس

<u>POWER</u>

power and energy calculations are important in circuit analysis.
Power is an indication of how much work (the conversion of energy from one form to another) can be done in a specified amount of time, that is, a *rate* of doing work. For instance, a large motor has more Power than a small motor because it can convert more electrical energy into mechanical energy in the same period of time. Since converted energy is measured in *joules* (J) and time in seconds (s), power is measured in joules/second (J/s). The electrical unit of measurement for power is the watt (W),

Power is the time rate of expending or absorbing energy, measured in watts (W).

$$P = \frac{W}{t}$$ (watts, W, or joules/second, J/s)

$$1 \text{ watt (W)} = 1 \text{ joule/second (J/s)}$$

$$1 \text{ horsepower} \cong 746 \text{ watts}$$

$$P = \frac{W}{t} = \frac{QV}{t} = V\frac{Q}{t}$$

But $I = \dfrac{Q}{t}$

so that $P = VI$ (watts)

$$P = VI = V\left(\frac{V}{R}\right)$$

and $P = \dfrac{V^2}{R}$ (watts)

or $P = VI = (IR)I$

and $P = I^2R$ (watts)

The magnitude of the power delivered or absorbed by a battery is given by

$P = EI$ (watts)

With E the battery terminal voltage and I the current through the source

EXAMPLE 5 Find the power delivered to the dc motor of Fig (1-18):

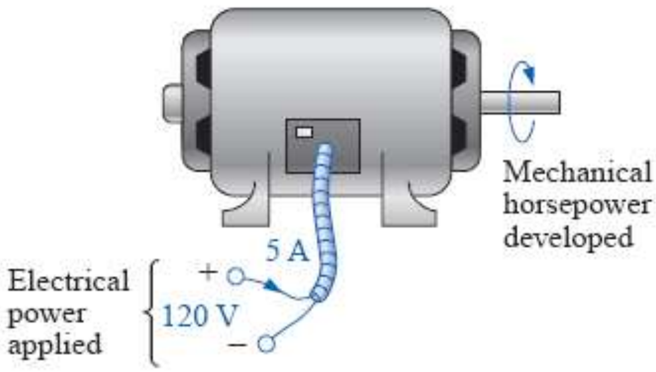


Fig.(1-18)

Sol.

$$P = VI = (120\ \text{V})(5\ \text{A}) = 600\ \text{W} = \mathbf{0.6\ kW}$$

EXAMPLE 6 what is the power dissipated by a 5Ω resistor if the current is 4 A?

Sol.

$$P = I^2R = (4\ \text{A})^2(5\ \Omega) = \mathbf{80\ W}$$

Energy

Energy (W) A quantity whose change in state is determined by the product of the rate of conversion (P) and the period involved (t). It is measured in joules (J) or watt seconds (Ws).

$$\boxed{W = Pt} \qquad \text{(wattseconds, Ws, or joules)}$$

$$\text{Energy (Wh)} = \text{power (W)} \times \text{time (h)}$$

$$\text{Energy (kWh)} = \frac{\text{power (W)} \times \text{time (h)}}{1000}$$

The kilowatt-hour meter is an instrument for measuring the energy

Example 7 How much energy (in kilowatt hours) is required to light a 60 W bulb continuously for 1 year (365 days)?
Sol.

$$W = \frac{Pt}{1000} = \frac{(60\text{ W})(24\text{ h/day})(365\text{ days})}{1000} = \frac{525{,}600\text{ Wh}}{1000}$$
$$= \mathbf{525.60\ kWh}$$

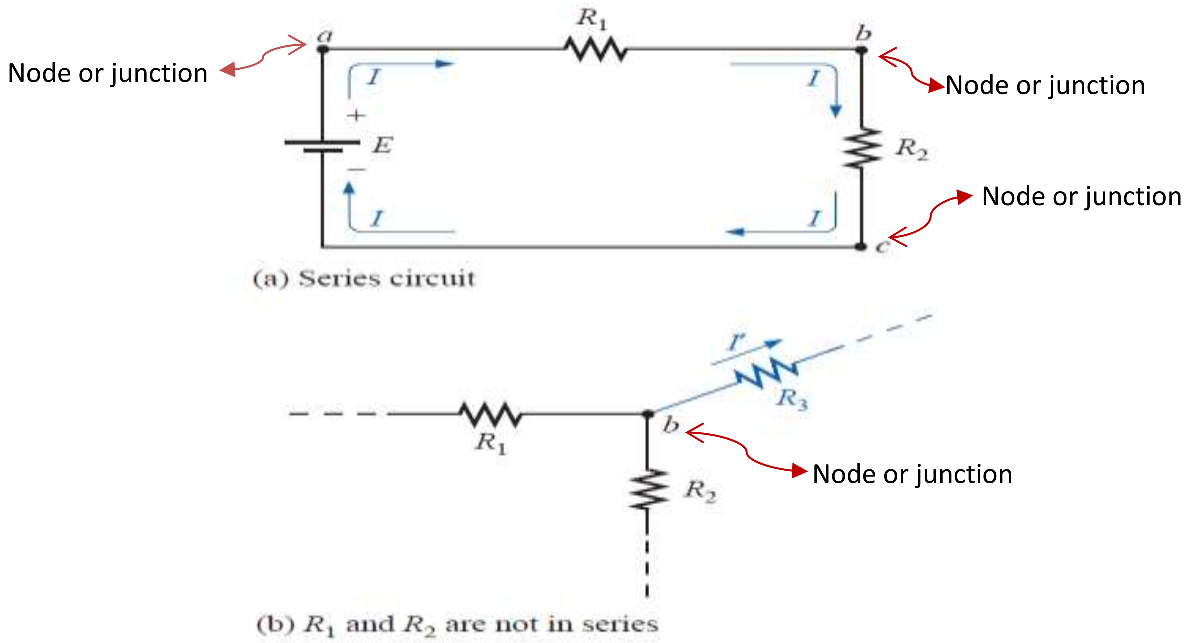
Example 8 How long can a 205 W television set be on before using more than 4 kWh of energy?

Sol.

$$W = \frac{Pt}{1000} \Rightarrow t\,(\text{hours}) = \frac{(W)(1000)}{P} = \frac{(4\text{ kWh})(1000)}{205\text{ W}} = \mathbf{19.51\ h}$$

2-1 SERIES CIRCUITS

A circuit consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Fig. 2-1 has three elements joined at three terminal points (*a*, *b*, and *c*) to provide a closed path for the current *I*.



Fig(2-1)
(a) Series circuit R1and R2 and E
(b) R_1 and R_2 and R_3 are **not** in series.

In Fig.(2-1) the resistors *R*1 and *R*2 are in series , the battery *E* and resistor *R*1 are in series, and the resistor *R*2 and the battery *E* are in series .Since all the elements are in series, the network is called a (**series circuit.)**

Note

1- The total resistance of a series circuit is the sum of the resistance levels.

$RT= R_1+R_2$ (ohm Ω)

2- The current is the same through each element and the current drawn from the source (Total current I_T) of Fig. (2-1a) equal:

$I=I_{R1}=I_{R2}=I_T$ (Amp)

I_T can be determined using Ohm's law.

$I= I_T=\frac{E}{RT}$ (Amp)

3- $V_1=IR_1$, $V_2=IR_2$, $V_T= V_1+V_2$ (Volt)

4- The power delivered to each resistor can then be determined using any one of three equations:

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1} \quad \text{(watts, W)}$$

The power delivered by the source is

$$P_{del} = EI \quad \text{(watts, W)}$$

The total power delivered to a resistive circuit is equal to the total power dissipated by the resistive elements.

$$P_{del} = P_1 + P_2 + P_3 + \cdots + P_N$$

EXAMPLE 1

a. Find the total resistance for the series circuit of Fig. (2-2)
b. Calculate the source current *Is*.
c. Determine the voltages V_1, V_2, and V_3.
d. Calculate the power dissipated by R_1, R_2, and R_3.
e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).

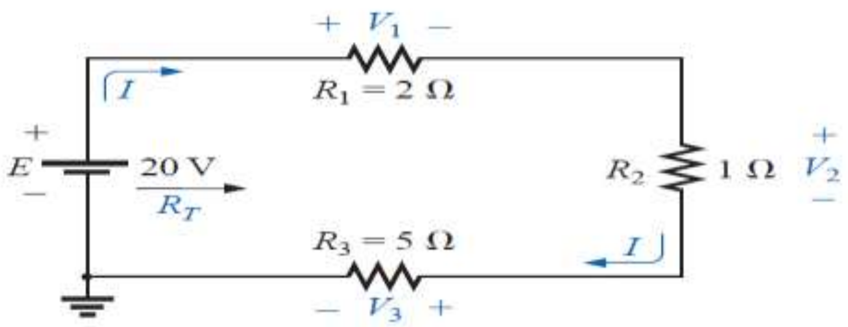


Fig (2-2)

Solutions:

a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\ \text{V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\ \text{A})(2\ \Omega) = \mathbf{5\ V}$
$V_2 = IR_2 = (2.5\ \text{A})(1\ \Omega) = \mathbf{2.5\ V}$
$V_3 = IR_3 = (2.5\ \text{A})(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1I_1 = (5\ \text{V})(2.5\ \text{A}) = \mathbf{12.5\ W}$
$P_2 = I_2^2R_2 = (2.5\ \text{A})^2(1\ \Omega) = \mathbf{6.25\ W}$
$P_3 = V_3^2/R_3 = (12.5\ \text{V})^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{del} = EI = (20\ \text{V})(2.5\ \text{A}) = \mathbf{50\ W}$
$P_{del} = P_1 + P_2 + P_3$
$50\ \text{W} = 12.5\ \text{W} + 6.25\ \text{W} + 31.25\ \text{W}$
$50\ \text{W} = 50\ \text{W}$ (checks)

EXAMPLE 2 : Determine R_T, I, and V_2 for the circuit of Fig.(2-3)

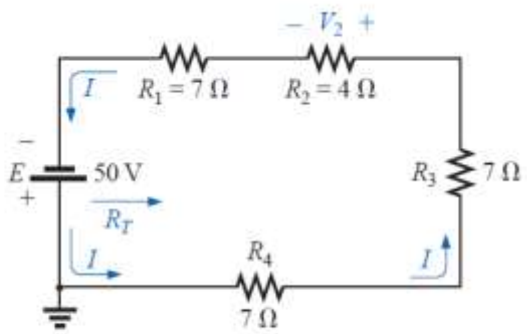


Fig.(2-3)

sol. Note the current direction as established by the battery and the polarity of the voltage drops across R_2 as determined by the current direction

$R_T=R_1+R_2+R_3+R_4$

$R_T=7+4+7+7=25\ \Omega$

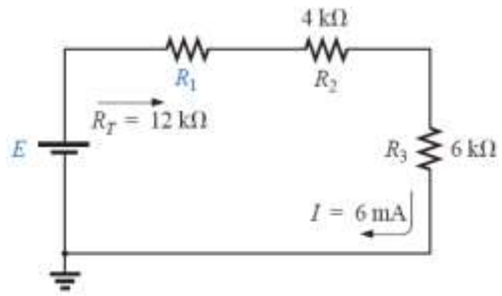
$$I = \frac{E}{R_T} = \frac{50\ \text{V}}{25\ \Omega} = \mathbf{2\ A}$$

$$V_2 = IR_2 = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$

EXAMPLE 3 Given R_T and I, calculate R_1 and E for the circuit of Fig.2-4 .

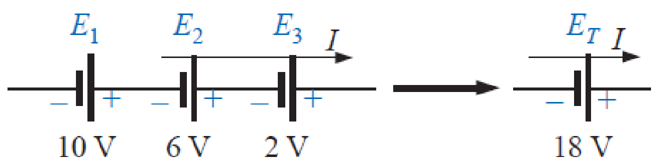
Solution:

$$R_T = R_1 + R_2 + R_3$$
$$12\ \text{k}\Omega = R_1 + 4\ \text{k}\Omega + 6\ \text{k}\Omega$$
$$R_1 = 12\ \text{k}\Omega - 10\ \text{k}\Omega = \mathbf{2\ k\Omega}$$
$$E = IR_T = (6 \times 10^{-3}\ \text{A})(12 \times 10^3\ \Omega) = \mathbf{72\ V}$$



Fig(2-4)

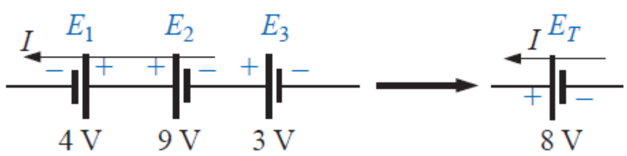
2-2 VOLTAGE SOURCES IN SERIES



(a)

$$ET = E_1 + E_2 + E_3 = 10\ \text{V} + 6\ \text{V} + 2\ \text{V} = 18\ \text{V}$$

and the polarity shown in the figure



(b)

$$ET = E2 + E3 - E1 = 9\ \text{V} + 3\ \text{V} - 4\ \text{V} = 8\ \text{V}$$

and the polarity shown in the figure

Fig (2.5)
(a ,b) Reducing series dc voltage sources to a single source.

2-3 KIRCHHOFF'S VOLTAGE LAW

Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.

A closed loop is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

the clockwise (CW) direction will be used throughout the text for all applications of Kirchhoff's voltage law. Be aware, however, that the same result will be obtained if the counterclockwise (CCW) direction is chosen and the law applied correctly. A plus sign is assigned to a potential rise (- to +), and a minus sign to a potential drop (+ to -). If we follow the current in Fig. (2-6) from point a, we first encounter a potential drop V1 (+ to -) across R1 and then another potential drop V2 across R2. Continuing through the voltage source, we have a potential rise E (- to +) before returning to point a. In symbolic form, where Σ represents (summation), the closed loop, and V the potential **drops** and **rises**, we have :

$$\Sigma_C V = 0$$ (Kirchhoff's voltage law in symbolic form)

Which for the circuit of Fig. (2-6) yields (clockwise direction, following the current I and starting at point d):

the applied voltage of a series circuit equals the sum of the voltage drops across the series elements.

$$+E - V_1 - V_2 = 0$$

$$E = V_1 + V_2$$

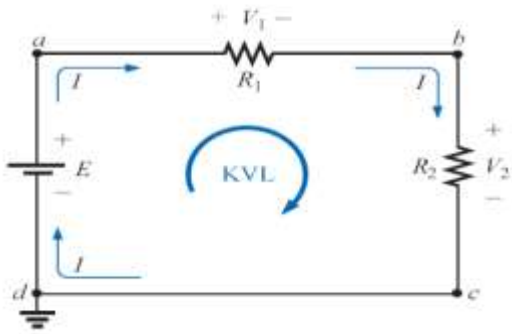


Fig (2-6) Applying Kirchhoff's voltage law to a series dc circuit.

Kirchhoff's voltage law can also be stated in the following form:

$$\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}}$$

<u>EXAMPLE 4</u> Determine the unknown voltages for the networks of Fig. (2-7)

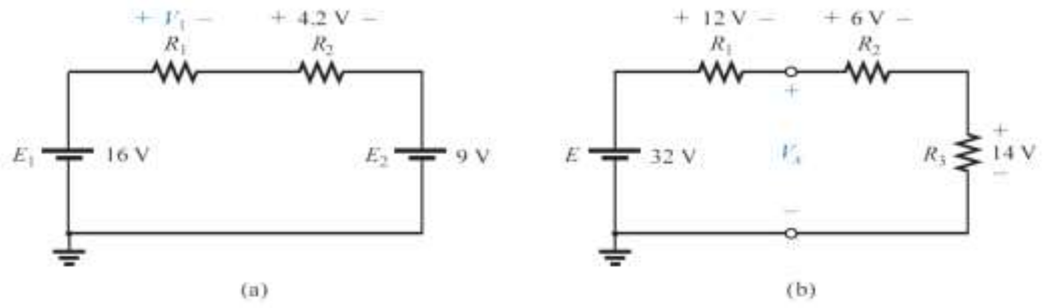


Fig (2-7)

Sol:

a-

$$+E_1 - V_1 - V_2 - E_2 = 0$$

and $$V_1 = E_1 - V_2 - E_2 = 16\text{ V} - 4.2\text{ V} - 9\text{ V}$$
$$= \mathbf{2.8\ V}$$

b-

$$+E - V_1 - V_x = 0$$

and $$V_x = E - V_1 = 32\text{ V} - 12\text{ V}$$
$$= \mathbf{20\ V}$$

or Using the clockwise direction for the other loop involving R_2 and R_3 will result in

$$+V_x - V_2 - V_3 = 0$$

and $$V_x = V_2 + V_3 = 6\text{ V} + 14\text{ V}$$
$$= \mathbf{20\ V}$$

EXAMPLE 5 For the circuit of Fig. (2-8)

a. Find R_T.
b. Find I.
c. Find V_1 and V_2.
d. Find the power to the 4-Ω and 6-Ω resistors.
e. Find the power delivered by the battery, and compare it to that dissipated by the 4-Ω and 6-Ω resistors combined.
f. Verify Kirchhoff's voltage law (clockwise direction).

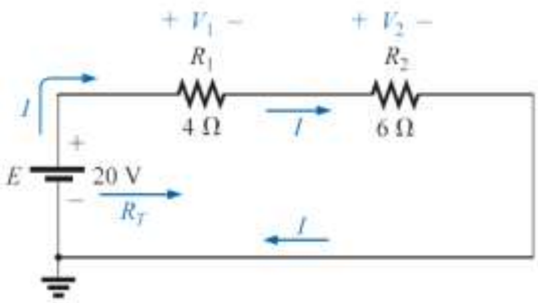


Fig (2-8)

Solutions:

a. $R_T = R_1 + R_2 = 4\ \Omega + 6\ \Omega = \mathbf{10\ \Omega}$

b. $I = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{10\ \Omega} = \mathbf{2\ A}$

c. $V_1 = IR_1 = (2\text{ A})(4\ \Omega) = \mathbf{8\ V}$
$V_2 = IR_2 = (2\text{ A})(6\ \Omega) = \mathbf{12\ V}$

d. $P_{4\Omega} = \dfrac{V_1^2}{R_1} = \dfrac{(8\text{ V})^2}{4} = \dfrac{64}{4} = \mathbf{16\ W}$

$P_{6\Omega} = I^2R_2 = (2\text{ A})^2(6\ \Omega) = (4)(6) = \mathbf{24\ W}$

e. $P_E = EI = (20\text{ V})(2\text{ A}) = \mathbf{40\ W}$
$P_E = P_{4\Omega} + P_{6\Omega}$
$40\text{ W} = 16\text{ W} + 24\text{ W}$
$40\text{ W} = 40\text{ W}$ (checks)

f. $\Sigma_{\circlearrowright} V = +E - V_1 - V_2 = 0$
$E = V_1 + V_2$
$20\text{ V} = 8\text{ V} + 12\text{ V}$
$20\text{ V} = 20\text{ V}$ (checks)

EXAMPLE 6 For the circuit of Fig. (2-9)

a. Determine V_2 using Kirchhoff's voltage law.
b. Determine I.
c. Find R_1 and R_3.

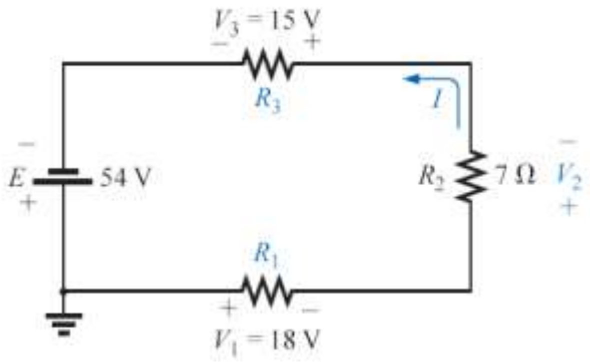


Fig. (2-9)

Solutions:

a. Kirchhoff's voltage law (clockwise direction):

$$-E + V_3 + V_2 + V_1 = 0$$

or $$E = V_1 + V_2 + V_3$$

and $$V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V} = \mathbf{21\text{ V}}$$

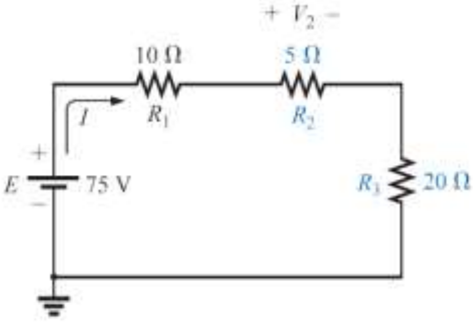
b. $$I = \frac{V_2}{R_2} = \frac{21\text{ V}}{7\ \Omega} = \mathbf{3\text{ A}}$$

c. $$R_1 = \frac{V_1}{I} = \frac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$$

$$R_3 = \frac{V_3}{I} = \frac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$$

2-4 INTERCHANGING SERIES ELEMENTS

The elements of a **series circuit** can be interchanged without affecting the total resistance, current, or power to each element. For instance, the network of Fig. (2-10) can be redrawn as shown in Fig.(2-11)



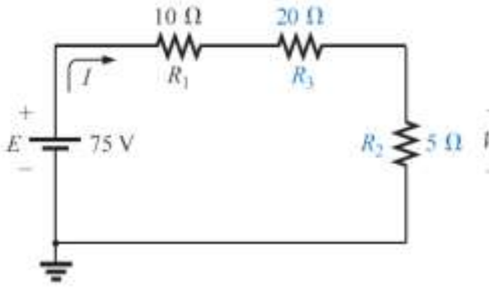


Fig. (2-10)

Series dc circuit with

elements to be interchanged

Fig. (2-11)

Circuit of

with R2and R3 interchanged.

EXAMPLE 7 Determine I and the voltage across the 7Ω resistor for the network of Fig. (2-12)

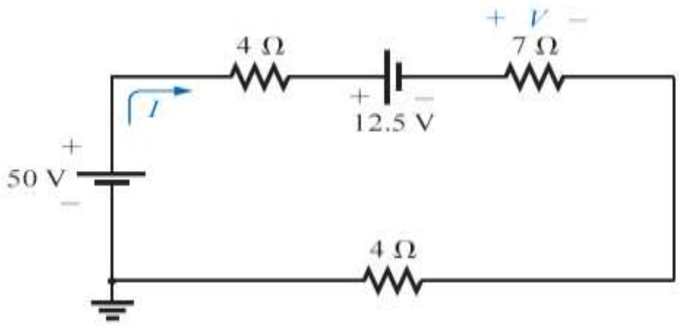


Fig (2-12)

Solution: The network is redrawn in Fig. (2-13)

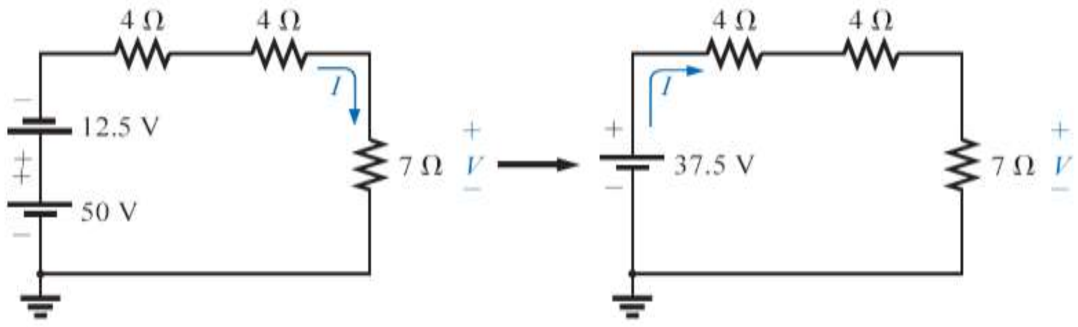


Fig (2-13)

$$R_T = (2)(4\ \Omega) + 7\ \Omega = 15\ \Omega$$

$$I = \frac{E}{R_T} = \frac{37.5\ \text{V}}{15\ \Omega} = \mathbf{2.5\ A}$$

$$V_{7\Omega} = IR = (2.5\ \text{A})(7\ \Omega) = \mathbf{17.5\ V}$$

2-5 VOLTAGE DIVIDER RULE (V.D.R.)

the voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.

$$V_x = \frac{R_x E}{R_T} \qquad \text{(voltage divider rule)}$$

EXAMPLE 8 Determine the voltage V1 for the network of Fig. (2-14)

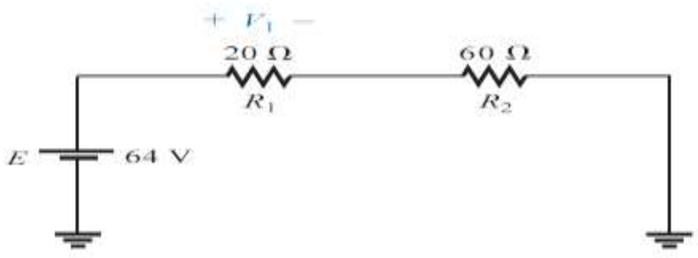


Fig (2-14)

Sol.

The circuit is simplified to fig (2-15)

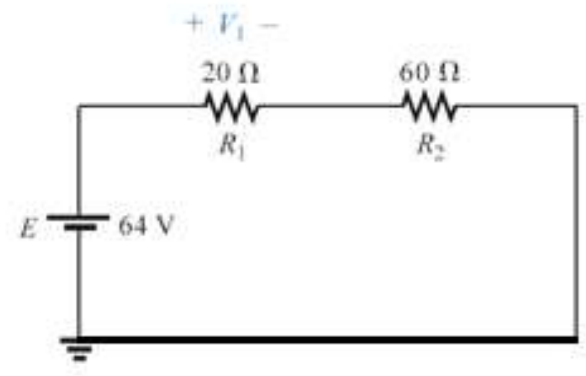


Fig (2-15)

$$V_1 = \frac{R_1 E}{R_T} = \frac{R_1 E}{R_1 + R_2} = \frac{(20\ \Omega)(64\ \text{V})}{20\ \Omega + 60\ \Omega} = \frac{1280\ \text{V}}{80} = \mathbf{16\ V}$$

EXAMPLE 9 Using the voltage divider rule, determine the voltages V1 and V3 for the series circuit of Fig. (2-16)

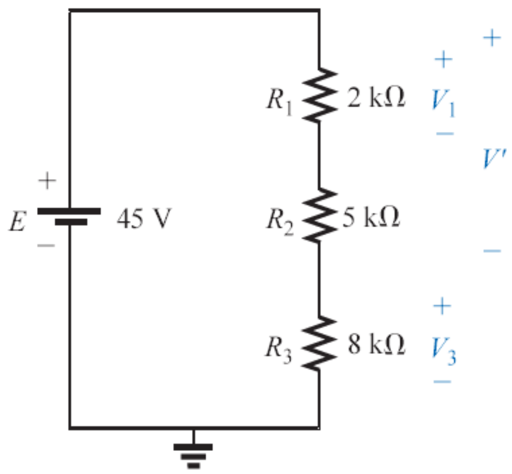


Fig (2-16)

Solution:

$$V_1 = \frac{R_1E}{R_T} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{2\ \text{k}\Omega + 5\ \text{k}\Omega + 8\ \text{k}\Omega} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega}$$

$$= \frac{(2 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega} = \frac{90\ \text{V}}{15} = \mathbf{6\ V}$$

$$V_3 = \frac{R_3E}{R_T} = \frac{(8\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(8 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega}$$

$$= \frac{360\ \text{V}}{15} = \mathbf{24\ V}$$

Note ***The rule can be extended to the voltage across two or more series elements.***

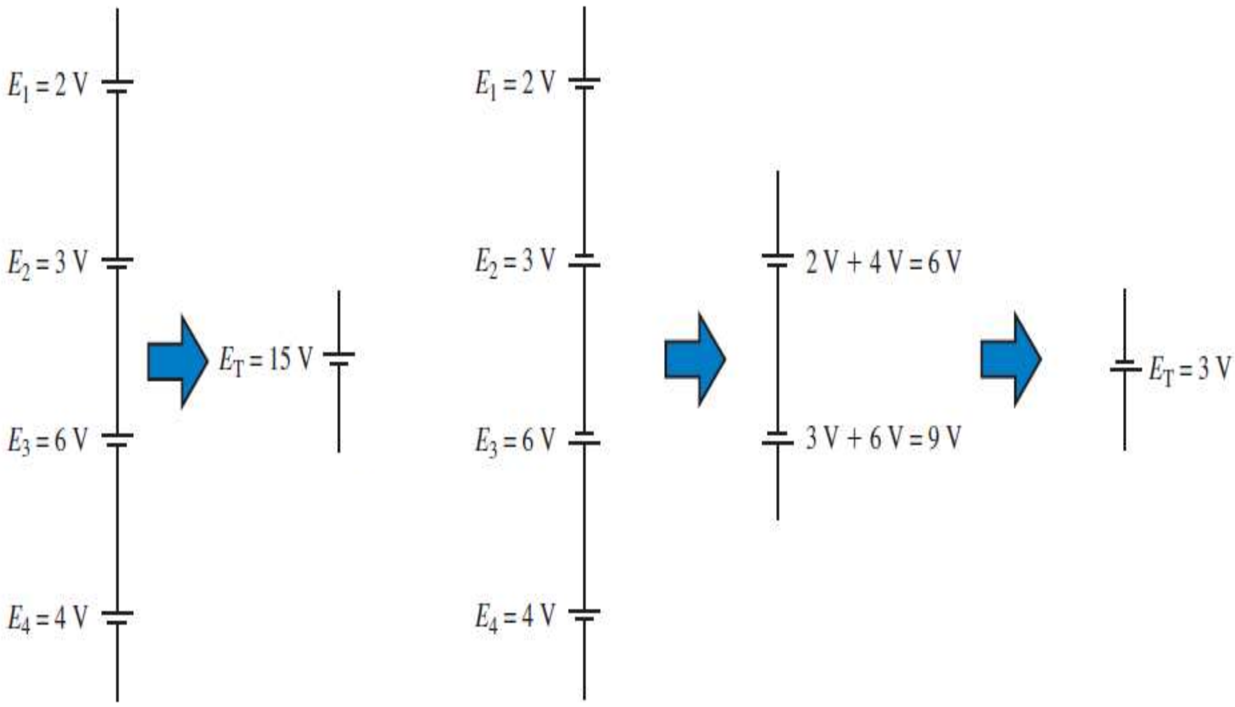
$$V' = \frac{R'E}{R_T} \qquad \text{(volts)}$$

<u>EXAMPLE 10</u> : Determine the voltage V′(by using voltage divider rule) in Fig.(2-16)across resistors R_1 and R_2.

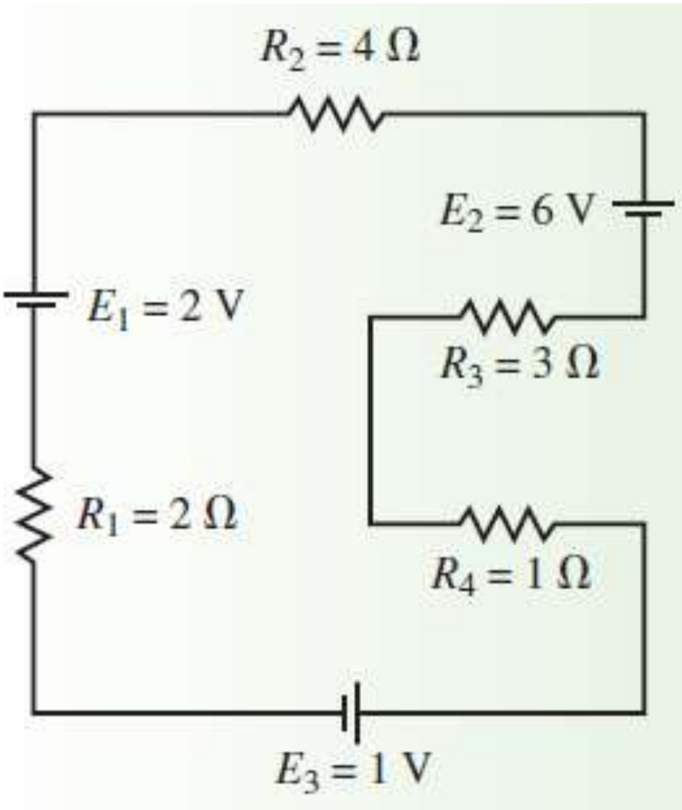
Solution:

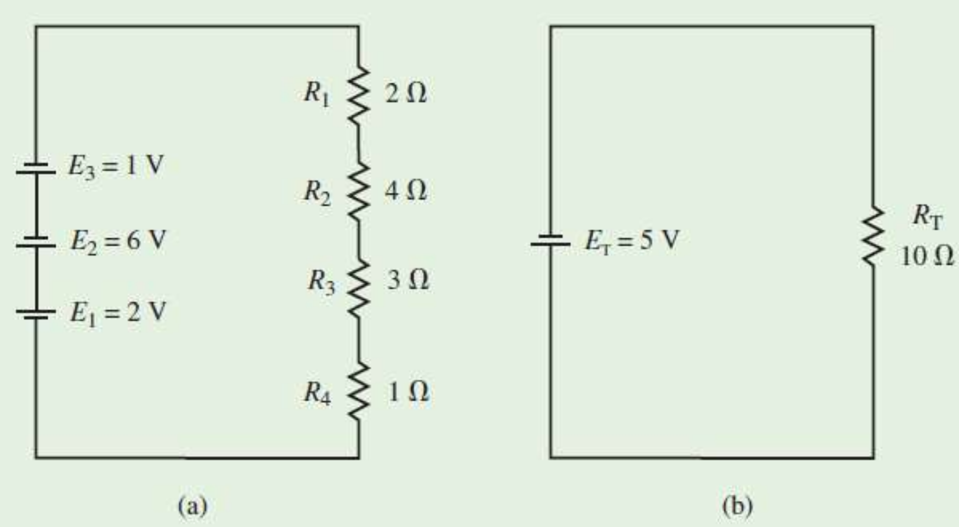
$$V' = \frac{R'E}{R_T} = \frac{(2\ \text{k}\Omega + 5\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(7\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \mathbf{21\ V}$$

Ex.



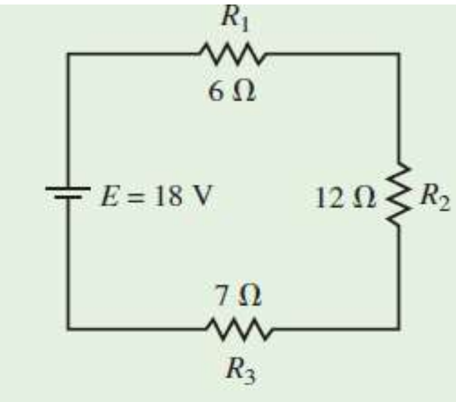
Ex. Find the total current(I_T):





$$I = \frac{E_T}{R_T} = \frac{6\text{ V} + 1\text{ V} - 2\text{ V}}{2\,\Omega + 4\,\Omega + 3\,\Omega + 1\,\Omega} = \frac{5\text{ V}}{10\,\Omega} = 0.500\text{ A}$$

Ex. Use the voltage divider rule to determine the voltage across each of the resistors in the circuit shown in Figure below. Show that the summation of voltage drops is equal to the applied voltage rise in the circuit.



Solution

$$R_T = 6\,\Omega + 12\,\Omega + 7\,\Omega = 25.0\,\Omega$$

$$V_1 = \left(\frac{6\,\Omega}{25\,\Omega}\right)(18\text{ V}) = 4.32\text{ V}$$

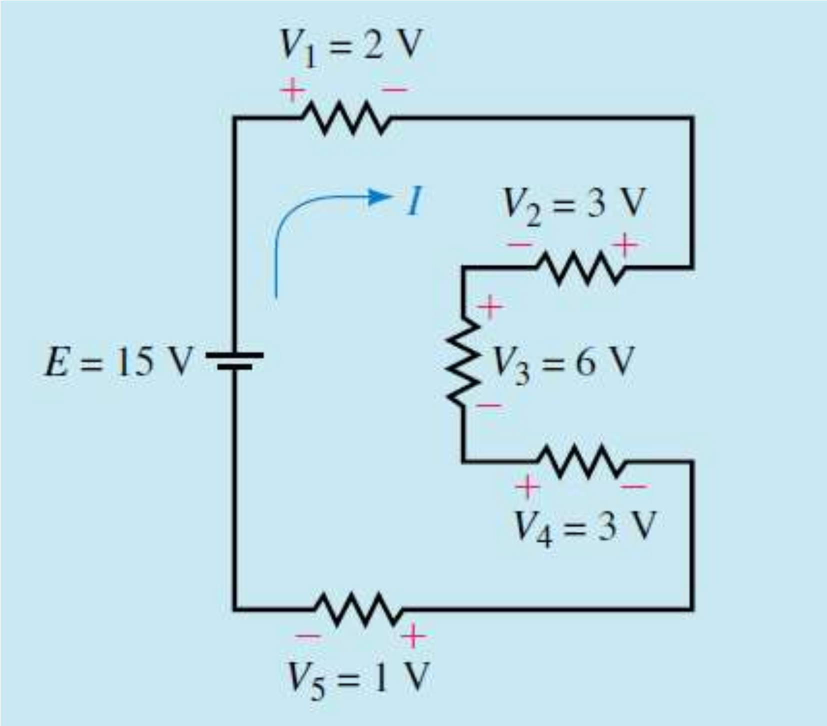
$$V_2 = \left(\frac{12\,\Omega}{25\,\Omega}\right)(18\text{ V}) = 8.64\text{ V}$$

$$V_3 = \left(\frac{7\,\Omega}{25\,\Omega}\right)(18\text{ V}) = 5.04\text{ V}$$

The total voltage drop is the summation

$$V_T = 4.32\text{ V} + 8.64\text{ V} + 5.04\text{ V} = 18.0\text{ V} = E$$

Ex.Verify Kirchhoff's voltage law for the circuit of Figure below:

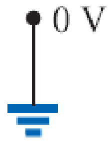


<u>Solution</u>: If we follow the direction of the current, we write the loop equation as:

$$15\text{ V} - 2\text{ V} - 3\text{ V} - 6\text{ V} - 3\text{ V} - 1\text{ V} = 0$$

Voltage Sources and Ground

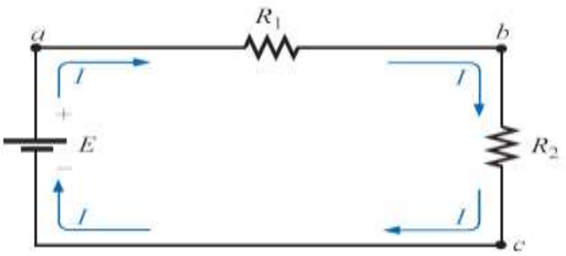
The symbol for the ground connection appears in Fig.(1) with its defined



Fig(1)

Ground potential

if we take the circuit of fig (2)



Fig(2)

If Fig.(2)is redrawn with a grounded supply, it might appear as shown in Fig. 3(a), (b), or (c). In any case, it is understood that the negative terminal of the battery and the bottom of the resistor R_2 are at ground potential.

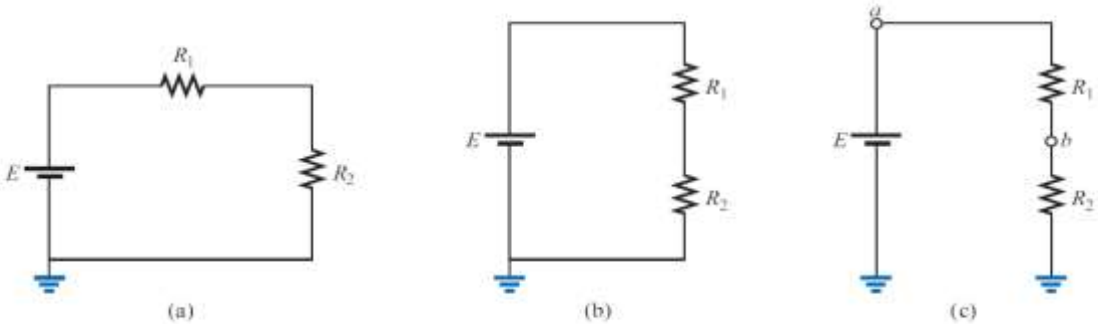


Fig (3)(a,b,c)

Three ways to sketch the same series dc circuit.

Example 1 Design a circuit by using the voltage divider rule of fig (4) such that ($V_{R1} = 4V_{R2}$).

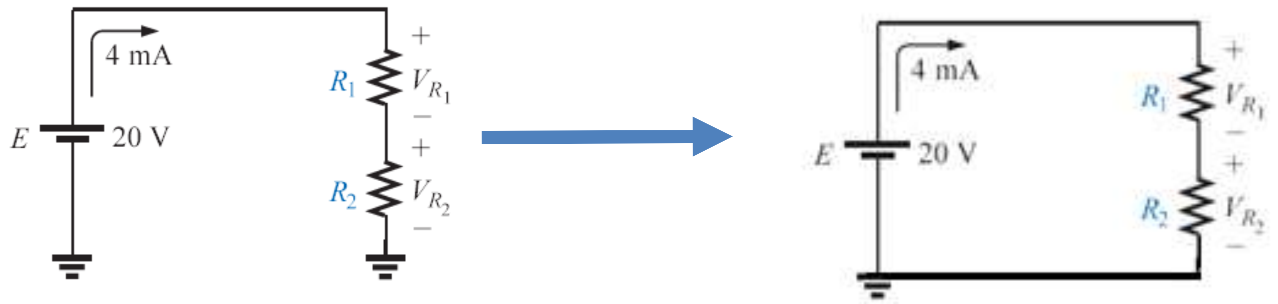


Fig.(4)

Solution: The total resistance is defined by :

$$R_T = \frac{E}{I} = \frac{20\text{ V}}{4\text{ mA}} = 5\text{ k}\Omega$$

$$V_{R1} = \frac{20\times R1}{R1+R2}$$

$$V_{R2} = \frac{20\times R2}{R1+R2}$$

Since $V_{R1} = 4\times V_{R2}$

$$\frac{20\times R1}{R1+R2} = 4 \times \frac{20\times R2}{R1+R2}$$

$$[\, R_1(\frac{20}{R1+R2}) = 4\, R_2(\frac{20}{R1+R2})] \div (\frac{20}{R1+R2})$$

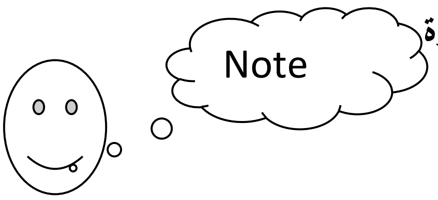
$$R_1 = 4R_2$$

Thus $$R_T = R_1 + R_2 = 4R_2 + R_2 = 5R_2$$

and $$5R_2 = 5\text{ k}\Omega$$
$$R_2 = \mathbf{1\text{ k}\Omega}$$

and $$R_1 = 4R_2 = \mathbf{4\text{ k}\Omega}$$

كلمة **design** تعني صمم والتصميم هنا معناه ايجاد القيم المجهولة بالدائرة



Voltage sources may be indicated as shown in Figs. 5(a) and 6(a) rather than as illustrated in Fig. 5(b) and 6(b)..

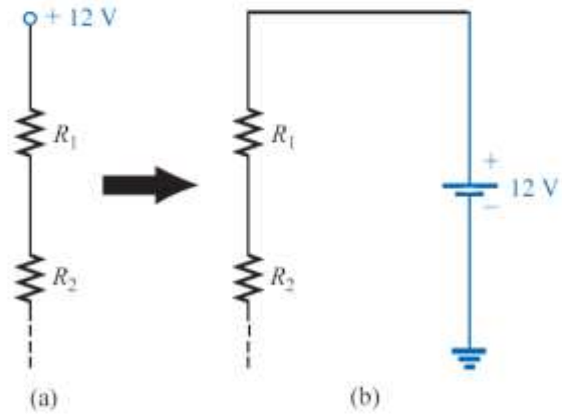
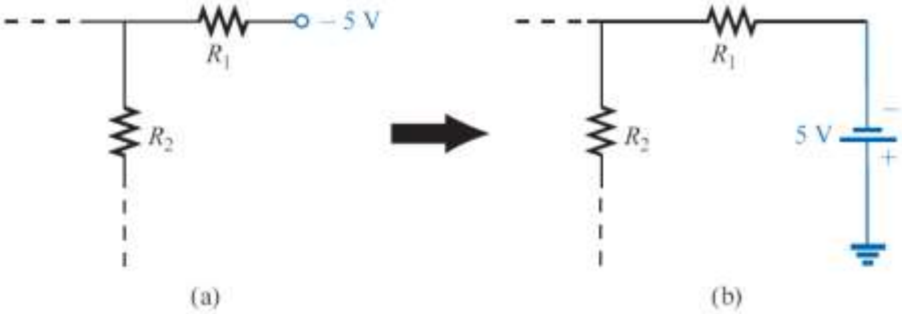


Fig (5)

Replacing the special notation for a positive dc voltage source with the standard symbol



Fig(6)

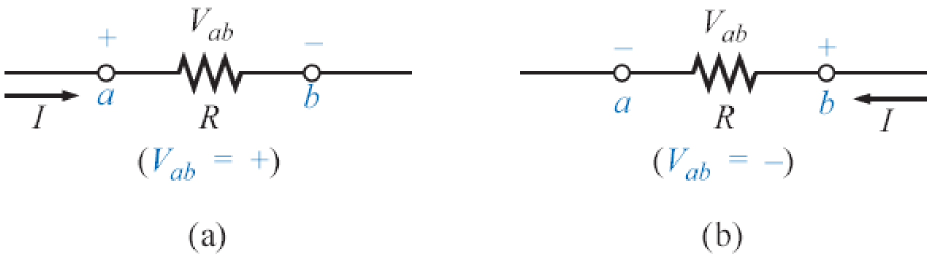
Replacing the notation for a negative dc supply with the standard symbol.

Double-Subscript Notation

The double-subscript notation V_{ab} specifies point (a) as the higher potential. If this is not the case, negative sign must be associated with the magnitude of V_{ab}.

In other words,

the voltage V_{ab} is the voltage at point(a) ***with respect to (w.r.t.)*** *point (b).*



Fig(7)

The fact that voltage is an across variable and exists between two points has resulted in a double-subscript notation that defines the first subscript as the higher potential. In Fig. 7(a), the two points that define the voltage across the resistor R are denoted by (a and b). Since (a) is the first subscript for V_{ab}, point (a) must have a higher potential than point (b)if V_{ab} is to have a positive value. If, point (b) is at a higher potential than point (a) ,V_{ab} will have a negative value, as indicated in Fig. 7(b).

Single-Subscript Notation

The single-subscript notation (V_a) specifies the voltage at point a with respect to ground (zero volts).

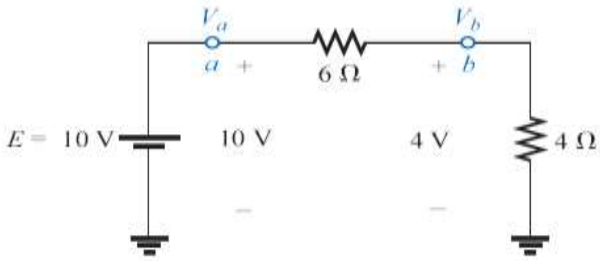


Fig.(8)

Defining the use of single-subscript notation

In Fig. 8, V_a is the voltage from point (a) to ground. In this case it is obviously 10 V since it is right across the source voltage E. The voltage V_b is the voltage from point (b) to ground. Because it is directly across the 4Ω resistor, V_b = 4 V.

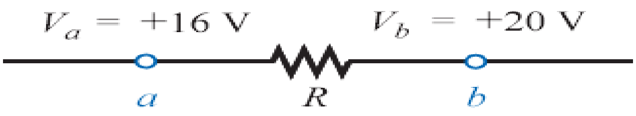
the following relationship exists:

$$V_{ab} = V_a - V_b \qquad (1)$$

So from the equation (1):

$$V_{ab} = V_a - V_b = 10\text{ V} - 4\text{ V}$$
$$= 6\text{ V}$$

<u>EXAMPLE 2</u> Find the voltage V_{ab} for the conditions of Fig.(9)



Fig(9)

Solution: Applying Eq.(1)

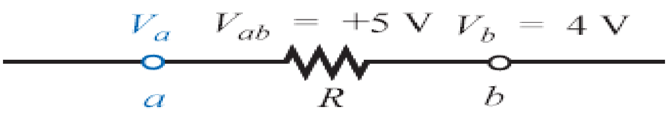
$V_{ab} = V_a - V_b = 16\text{ V} - 20\text{ V}$

$= -4\text{ V}$

<u>Note</u>: the negative sign means that point (b) is at a higher potential than point (a).

<u>EXAMPLE 3</u> Find the voltage V_a for the configuration of Fig.(10)

Solution: Applying Eq. (1):



Fig(10)

$V_{ab} = V_a - V_b$

and $V_a = V_{ab} + V_b = 5\text{ V} + 4\text{ V}$

$= 9\text{ V}$

EXAMPLE 4 Find the voltage V_{ab} for the configuration of Fig.(11)

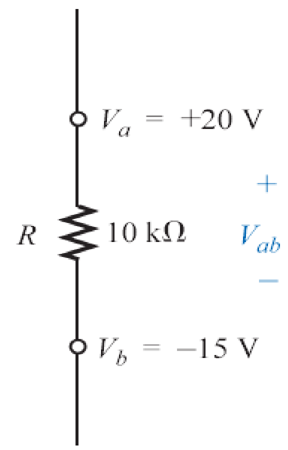


Fig (11)

Solution: Applying Eq. (1):

$V_{ab} = V_a - V_b = 20\ V - (-15\ V) = 20\ V + 15\ V$

$= 35\ V$

EXAMPLE 5 Find the voltages V_a, V_b, V_c, and V_{ac} for the network of Fig(12).

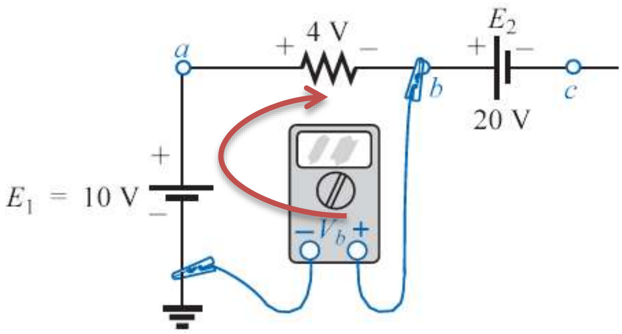


Fig (12)

Solution: Starting at ground potential (zero volts), we proceed through a rise of 10 V to reach point (a) and then pass through(a) drop in potential of 4 V to point (b). The result is that the meter will read :

Va=10 V

To find V_b we use Kirchhoff's voltage law :

$+E_1 - 4 - V_b = 0$

$10 - 4 = V_b$

$V_b = 6$ V

$V_{bc} = V_b - V_c$

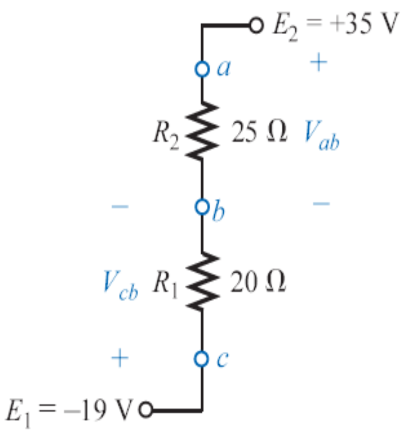
$V_c = V_b - V_{bc}$

$V_c = V_b - 20$ V = 6 V - 20 V = -14 V

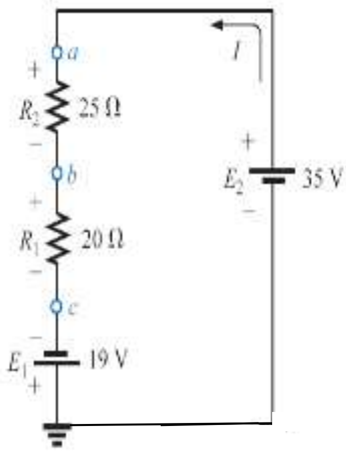
$V_{ac} = V_a - V_c = 10$ V - (-14 V)

= 24 V

EXAMPLE 6 Determine V_{ab}, V_{cb}, and V_c for the network of Fig.(13)



Redraw the network as shown in Fig. (14)



Fig(13)

Fig(14)

<u>Sol</u>:

$E_T = 19+35 = 54$ V

$R_T = 20+25 = 45$

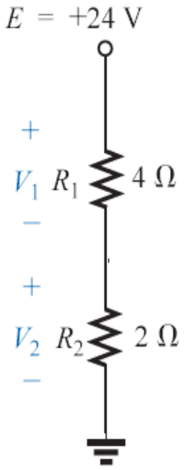
$$I = \frac{54\text{ V}}{45\ \Omega} = 1.2\text{ A}$$

$$V_{ab} = IR_2 = (1.2\text{ A})(25\ \Omega) = \mathbf{30\ V}$$

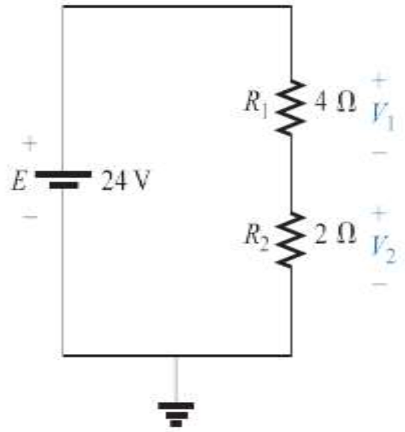
$$V_{cb} = -IR_1 = -(1.2\text{ A})(20\ \Omega) = \mathbf{-24\ V}$$

$$V_c = E_1 = \mathbf{-19\ V}$$

EXAMPLE 7 Using the voltage divider rule, determine the voltagesV1 and V2 of Fig.(15).



Redraw the network as shown in Fig. (16)

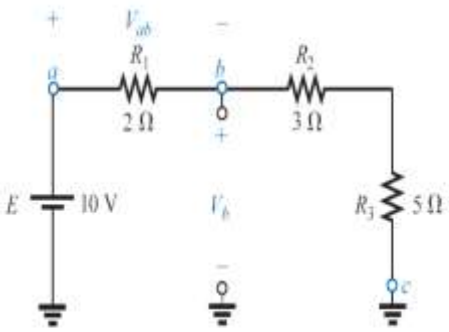


Fig(15) fig (16)

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(4\ \Omega)(24\ \text{V})}{4\ \Omega + 2\ \Omega} = \mathbf{16\ V}$$

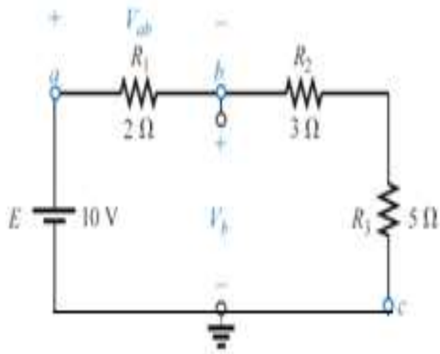
$$V_2 = \frac{R_2 E}{R_1 + R_2} = \frac{(2\ \Omega)(24\ \text{V})}{4\ \Omega + 2\ \Omega} = \mathbf{8\ V}$$

EXAMPLE 8 For the network of Fig. (17)



a

Redraw the network as shown in Fig. (17 b)



b

Fig(17)

a. Calculate V_{ab}.
b. Determine V_b.
c. Calculate V_c.

Solutions:

a. Voltage divider rule:

$$V_{ab} = \frac{R_1 E}{R_T} = \frac{(2\ \Omega)(10\ \text{V})}{2\ \Omega + 3\ \Omega + 5\ \Omega} = \mathbf{+2\ V}$$

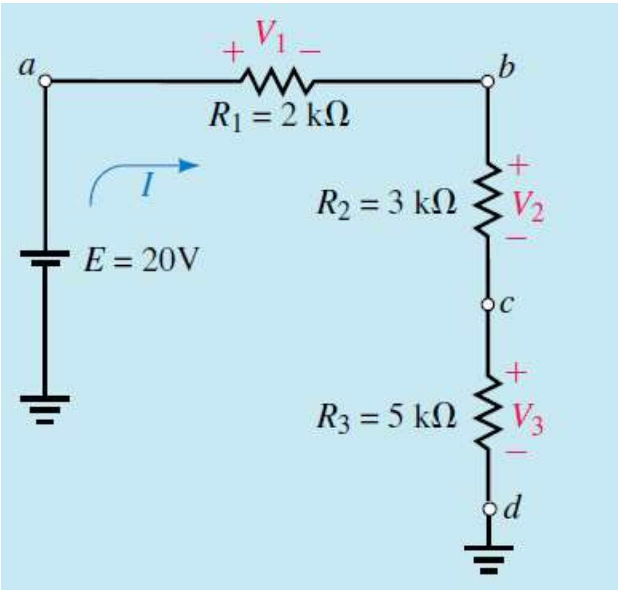
b. Voltage divider rule:

$$V_b = V_{R_2} + V_{R_3} = \frac{(R_2 + R_3)E}{R_T} = \frac{(3\ \Omega + 5\ \Omega)(10\ \text{V})}{10\ \Omega} = \mathbf{8\ V}$$

or $$V_b = V_a - V_{ab} = E - V_{ab} = 10\ \text{V} - 2\ \text{V} = \mathbf{8\ V}$$

c. V_c = ground potential = **0 V**

Ex. For the circuit of Figure below determine the voltages *Va*



Solution Applying the voltage divider rule, we determine the voltage across each resistor as follows:

$$V_1 = \frac{2\ k\Omega}{2\ k\Omega + 3\ k\Omega + 5\ k\Omega}(20\ V) = 4.00\ V$$

$$V_2 = \frac{3\ k\Omega}{2\ k\Omega + 3\ k\Omega + 5\ k\Omega}(20\ V) = 6.00\ V$$

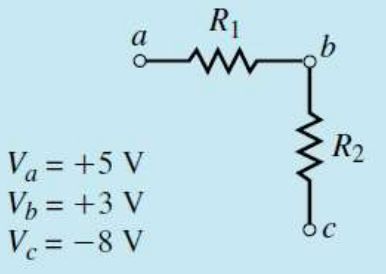
$$V_3 = \frac{5\ k\Omega}{2\ k\Omega + 3\ k\Omega + 5\ k\Omega}(20\ V) = 10.00\ V$$

Now we solve for the voltage at each of the points as follows:

$$V_a = 4\ V + 6\ V + 10\ V = +20\ V = E$$
$$V_b = 6\ V + 10\ V = +16.0\ V$$
$$V_c = +10.0\ V$$
$$V_d = 0\ V$$

Ex. For the circuit of Figure below, determine the voltages Vab and Vcb given that:

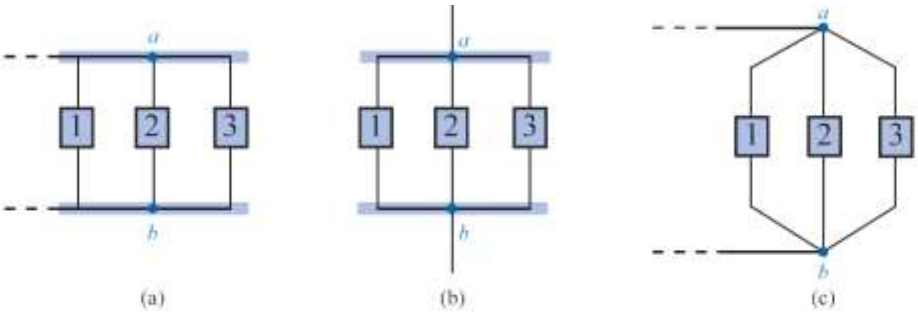
$V_a = +5\ V$, $V_b = +3\ V$, and $V_c = -8\ V$.



$$V_{ab} = +5\ V - (+3\ V) = +2\ V$$
$$V_{cb} = -8\ V - (+3\ V) = -11\ V$$

PARALLEL RESISTANCE

Two elements, branches, or networks are in parallel if they have two points in common.



Different ways of three parallel elements.

1- TOTAL CONDUCTANCE AND RESISTANCE

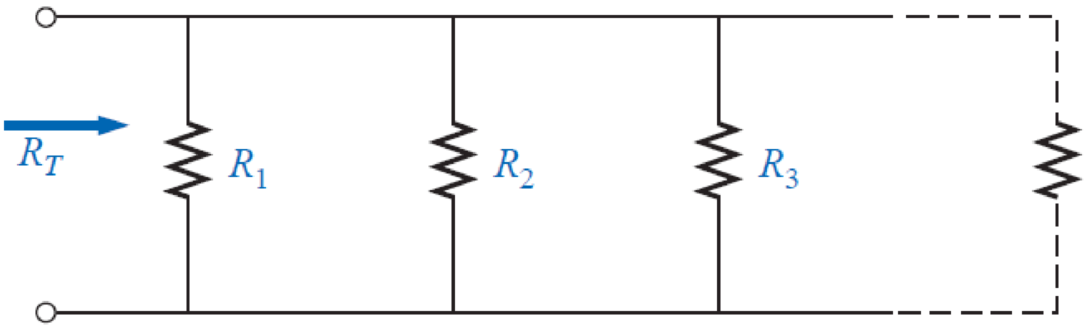


Fig.(1) The total resistance of parallel resistors

Since $G = 1/R$,
For parallel elements in fig.(1), the total conductance is the sum of the individual conductance.
Fig.(2) illustrate parallel connection.

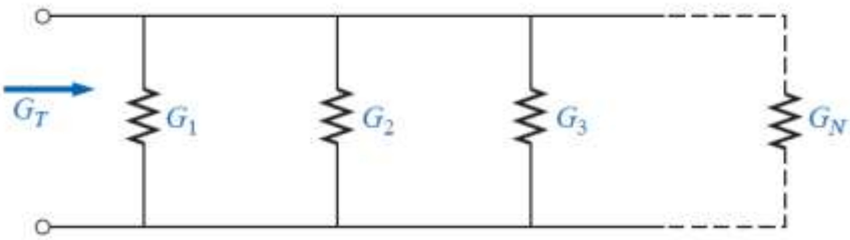
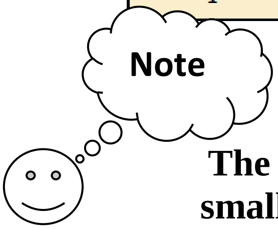


Fig.(2)

That is, for the parallel network of Fig. (2) we write

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$



The total resistance of parallel resistors is always less than the value of the smallest resistor.

the total resistance of two parallel resistors is the product of the two divided by their sum.

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

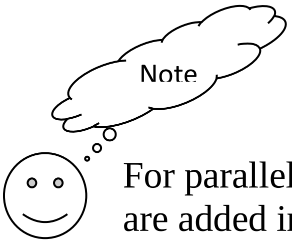
For three resistors we can also be expanded the form of above to:

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

For *equal* resistors in parallel, the equation becomes significantly easier to apply
For *N* equal resistors in parallel:

$$R_T = \frac{R}{N}$$

2- **The voltage across parallel elements is the same.**
3- **the source current (Is) is equal to the sum of the individual branch currents.**



For parallel resistors, the total resistance will always decrease as additional elements are added in parallel.

<u>Examples</u>

<u>EX.(1)</u>

a. Find the total resistance of the network of Fig. (3)

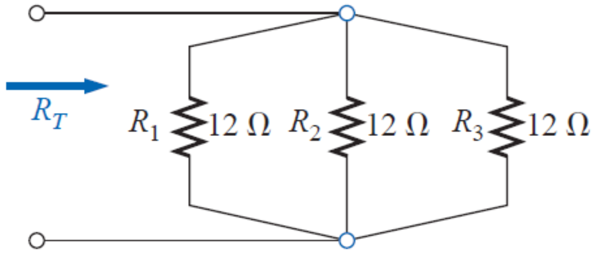
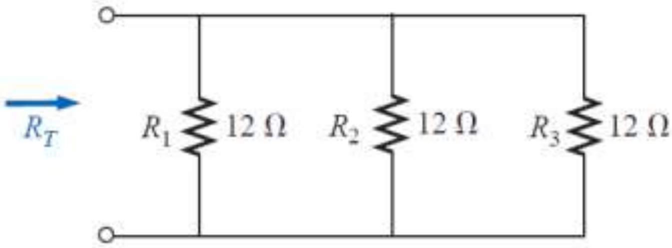


Fig.(3)

<u>solutions</u>:



$$R_T = \frac{R}{N} = \frac{12\ \Omega}{3} = \mathbf{4\ \Omega}$$

b. Calculate the total resistance for the network of Fig.(4)

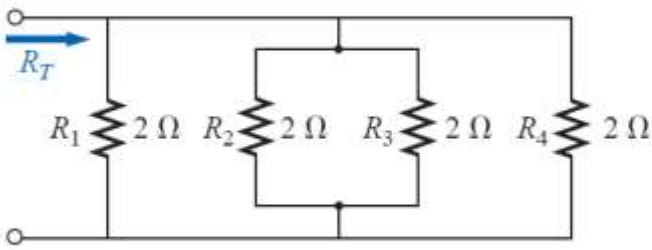
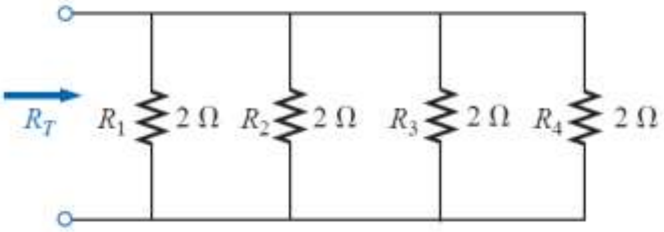


fig.(4)



$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = \mathbf{0.5\ \Omega}$$

Ex.2

Determine the values of $R1$, $R2$, and $R3$ in Fig(5) $R2 = 2R1$ and $R3 = 2R2$ and the total resistance is 16 kΩ.

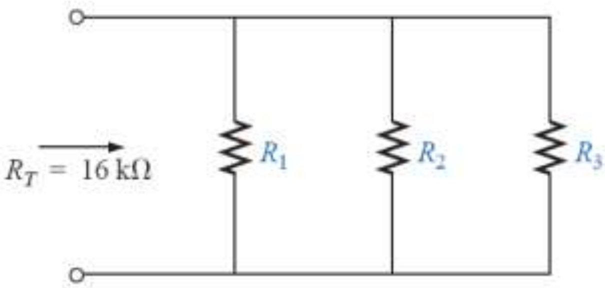


Fig.(5)

Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{16\text{ k}\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1}$$

since $\qquad R_3 = 2R_2 = 2(2R_1) = 4R_1$

and $\qquad \frac{1}{16\text{ k}\Omega} = \frac{1}{R_1} + \frac{1}{2}\left(\frac{1}{R_1}\right) + \frac{1}{4}\left(\frac{1}{R_1}\right)$

$$\frac{1}{16\text{ k}\Omega} = 1.75\left(\frac{1}{R_1}\right)$$

with $\qquad R_1 = 1.75(16\text{ k}\Omega) = \mathbf{28\text{ k}\Omega}$

R2= 2R1
R2= 2× **28= 36 KΩ**
R3=2×**36= 72 KΩ**

<u>Ex.3</u> For the parallel network of Fig. (6)

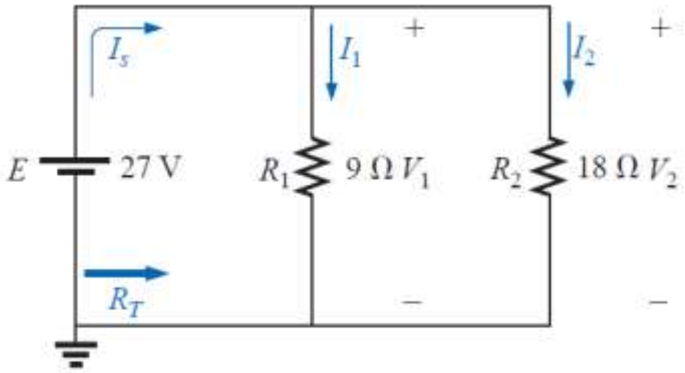


Fig.(6)

a. Calculate R_T.
b. Determine I_s.
c. Calculate I_1 and I_2, and demonstrate that $I_s = I_1 + I_2$.
d. Determine the power to each resistive load.
e. Determine the power delivered by the source, and compare it to the total power dissipated by the resistive elements.

Solutions:

a. $$R_T = \frac{R_1R_2}{R_1 + R_2} = \frac{(9\ \Omega)(18\ \Omega)}{9\ \Omega + 18\ \Omega} = \frac{162\ \Omega}{27} = \mathbf{6\ \Omega}$$

b. $$I_s = \frac{E}{R_T} = \frac{27\text{ V}}{6\ \Omega} = \mathbf{4.5\ A}$$

c. $$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27\text{ V}}{9\ \Omega} = \mathbf{3\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27\text{ V}}{18\ \Omega} = \mathbf{1.5\ A}$$

$$I_s = I_1 + I_2$$

$$4.5\text{ A} = 3\text{ A} + 1.5\text{ A}$$

$$\mathbf{4.5\ A} = \mathbf{4.5\ A} \quad \text{(checks)}$$

d. $P_1 = V_1I_1 = EI_1 = (27\text{ V})(3\text{ A}) = \mathbf{81\text{ W}}$
$P_2 = V_2I_2 = EI_2 = (27\text{ V})(1.5\text{ A}) = \mathbf{40.5\text{ W}}$

e. $P_s = EI_s = (27\text{ V})(4.5\text{ A}) = \mathbf{121.5\text{ W}}$
$= P_1 + P_2 = 81\text{ W} + 40.5\text{ W} = \mathbf{121.5\text{ W}}$

EX.(4)

Given the information provided in Fig. (7)

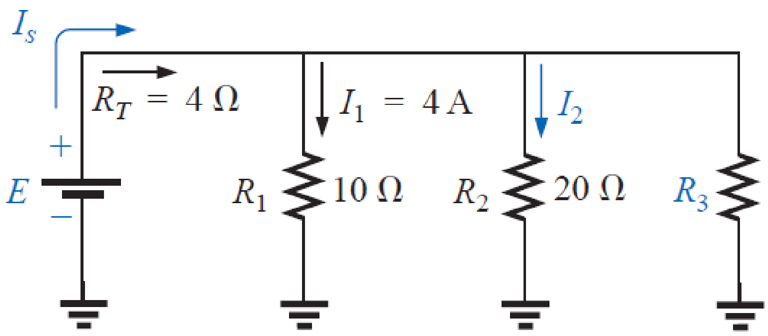


Fig.(7)

Sol.

a. Determine R_3.
b. Calculate E.
c. Find I_s.
d. Find I_2.
e. Determine P_2.

Solutions:

a. $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{4\ \Omega} = \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega} + \frac{1}{R_3}$$

$$0.25\text{ S} = 0.1\text{ S} + 0.05\text{ S} + \frac{1}{R_3}$$

$$0.25\text{ S} = 0.15\text{ S} + \frac{1}{R_3}$$

$$\frac{1}{R_3} = 0.1\text{ S}$$

$$R_3 = \frac{1}{0.1\text{ S}} = \mathbf{10\ \Omega}$$

b. $E = V_1 = I_1R_1 = (4\text{ A})(10\ \Omega) = \mathbf{40\ V}$

c. $I_s = \dfrac{E}{R_T} = \dfrac{40\text{ V}}{4\ \Omega} = \mathbf{10\ A}$

d. $I_2 = \dfrac{V_2}{R_2} = \dfrac{E}{R_2} = \dfrac{40\text{ V}}{20\ \Omega} = \mathbf{2\ A}$

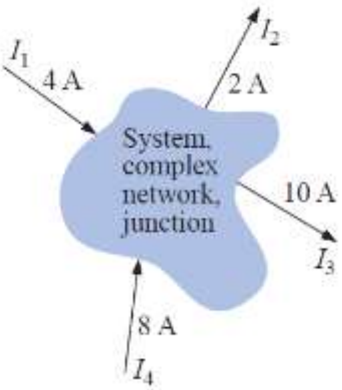
e. $P_2 = I_2^2R_2 = (2\text{ A})^2(20\ \Omega) = \mathbf{80\ W}$

KIRCHHOFF'S CURRENT LAW

Kirchhoff's current law (KCL) states that the algebraic sum of the currents entering and leaving an area, system, or junction is zero.

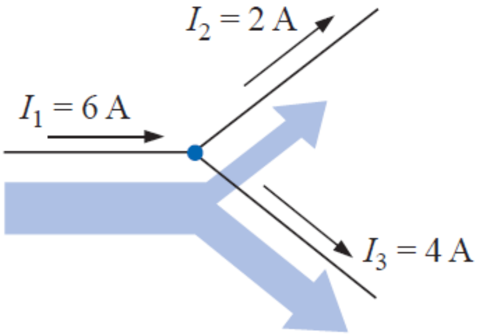
In other word

the sum of the currents entering an area, system, or junction must equal the sum of the currents leaving the area, system, or junction. see figure (8)&(9)



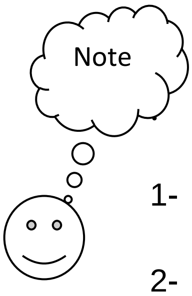
$$I_1 + I_4 = I_2 + I_3$$
$$4\text{ A} + 8\text{ A} = 2\text{ A} + 10\text{ A}$$
$$12\text{ A} = 12\text{ A}$$

Fig.(8)Introducing Kirchhoff's current law



$$\Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$
$$6\text{ A} = 2\text{ A} + 4\text{ A}$$
$$6\text{ A} = 6\text{ A} \quad \text{(checks)}$$

fig(9) Demonstrating Kirchhoff's current law.



1- **<u>A branch</u>** represents a single element such as a voltage source or a resistor.

2- **<u>A node</u>** or **<u>a junction</u>** the point of connection between two or more branches

<u>EX.(5)</u>

Determine the currents *I*3 and *I*5 of Fig. (10) through applications of Kirchhoff's current law.

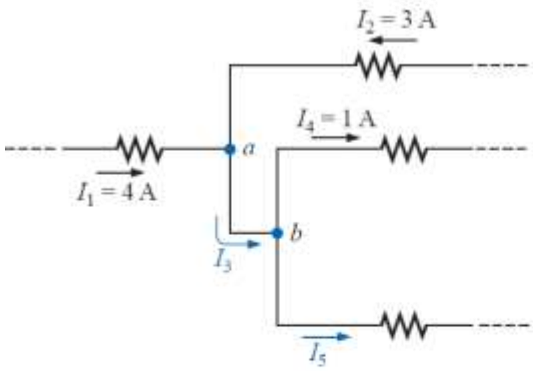


Fig.(10)

<u>Sol.</u>

For node a,

$$I_1 + I_2 = I_3$$
$$4\text{ A} + 3\text{ A} = I_3$$

and $$I_3 = \mathbf{7\text{ A}}$$

For node b,

$$I_3 = I_4 + I_5$$
$$7\text{ A} = 1\text{ A} + I_5$$

and $$I_5 = 7\text{ A} - 1\text{ A} = \mathbf{6\text{ A}}$$

EX.(6)

Find the magnitude and direction of the currents *I*3, *I*4, *I*6, and *I*7 for the network of Fig. (11). Even though the elements are not in series or parallel, Kirchhoff's current law can be applied to determine all the unknown currents.

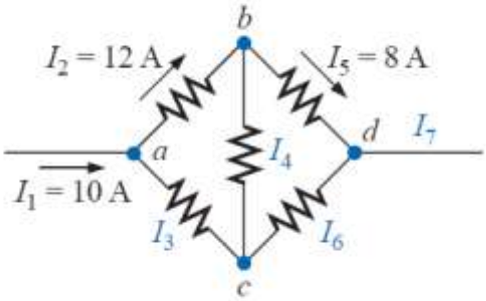


Fig.(11)

Solution: Considering the overall system, we know that the current entering must equal that leaving. Therefore,

$$I_7 = I_1 = \mathbf{10\ A}$$

Since 10 A are entering node *a* and 12 A are leaving, I_3 must be supplying current to the node.

Applying Kirchhoff's current law at node *a*,

$$I_1 + I_3 = I_2$$

$$10\text{ A} + I_3 = 12\text{ A}$$

and

$$I_3 = 12\text{ A} - 10\text{ A} = \mathbf{2\ A}$$

At node *b*, since 12 A are entering and 8 A are leaving, I_4 must be leaving. Therefore,

$$I_2 = I_4 + I_5$$

$$12\text{ A} = I_4 + 8\text{ A}$$

and

$$I_4 = 12\text{ A} - 8\text{ A} = \mathbf{4\ A}$$

At node c, I_3 is leaving at 2 A and I_4 is entering at 4 A, requiring that I_6 be leaving. Applying Kirchhoff's current law at node c,

$$I_4 = I_3 + I_6$$

$$4\text{ A} = 2\text{ A} + I_6$$

and

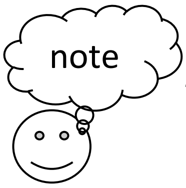
$$I_6 = 4\text{ A} - 2\text{ A} = \mathbf{2\text{ A}}$$

As a check at node d,

$$I_5 + I_6 = I_7$$

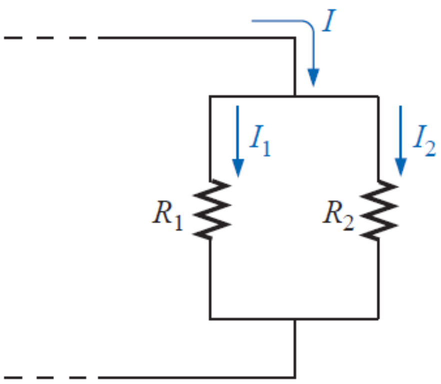
$$8\text{ A} + 2\text{ A} = 10\text{ A}$$

$$\mathbf{10\text{ A}} = \mathbf{10\text{ A}} \quad \text{(checks)}$$



1-For two parallel elements of equal value, the current will divide equally.
2-For parallel elements with different values, the smaller the resistance, the greater the share of input current.

CURRENT DIVIDER RULE



Note difference in subscripts.

$$I_1 = \frac{R_2 I}{R_1 + R_2}$$

$$I_2 = \frac{R_1 I}{R_1 + R_2}$$

EX.(7)

Find the current $I1$ for the network of Fig.(12)

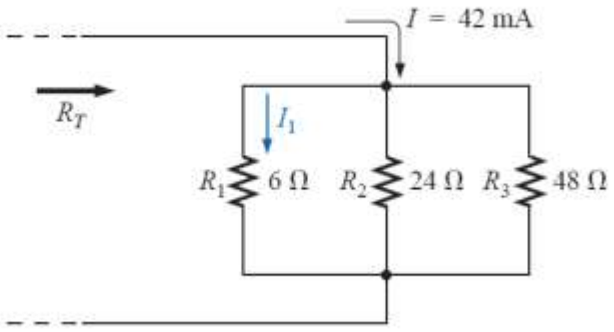


Fig.(12)

Sol.

$$\frac{1}{R_T} = \frac{1}{6\ \Omega} + \frac{1}{24\ \Omega} + \frac{1}{48\ \Omega} = 0.1667\ \text{S} + 0.0417\ \text{S} + 0.0208\ \text{S}$$
$$= 0.2292\ \text{S}$$

$$24\ \Omega \parallel 48\ \Omega = \frac{(24\ \Omega)(48\ \Omega)}{24\ \Omega + 48\ \Omega} = 16\ \Omega$$

$$I_1 = \frac{16\ \Omega(42\ \text{mA})}{16\ \Omega + 6\ \Omega} = \mathbf{30.54\ mA}$$

EX.(8)

Determine the magnitude of the currents $I1$, $I2$, and $I3$ for the network of Fig.(13)

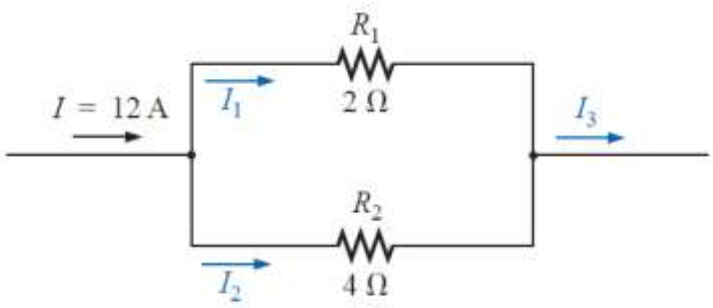


Fig.(13)

Sol.

$$I_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4\,\Omega)(12\,\text{A})}{2\,\Omega + 4\,\Omega} = \mathbf{8\,A}$$

Applying Kirchhoff's current law,

$$I = I_1 + I_2$$

and $$I_2 = I - I_1 = 12\,\text{A} - 8\,\text{A} = \mathbf{4\,A}$$

or, using the current divider rule again,

$$I_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(2\,\Omega)(12\,\text{A})}{2\,\Omega + 4\,\Omega} = \mathbf{4\,A}$$

The total current entering the parallel branches must equal that leaving. Therefore,

$$I_3 = I = \mathbf{12\,A}$$

or $$I_3 = I_1 + I_2 = 8\,\text{A} + 4\,\text{A} = \mathbf{12\,A}$$

Ex.(9)

Determine the resistance $R1$ to effect the division of current in Fig.(14)

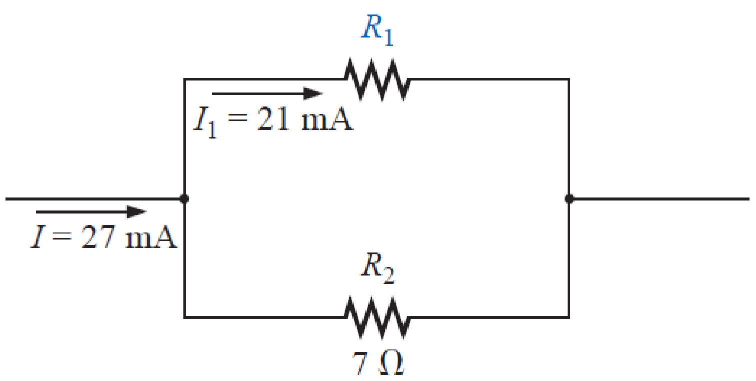


Fig.(14)

Solution: Applying the current divider rule,

$$I_1 = \frac{R_2 I}{R_1 + R_2}$$

and
$$(R_1 + R_2)I_1 = R_2 I$$
$$R_1 I_1 + R_2 I_1 = R_2 I$$
$$R_1 I_1 = R_2 I - R_2 I_1$$
$$R_1 = \frac{R_2(I - I_1)}{I_1}$$

Substituting values:

$$R_1 = \frac{7\ \Omega(27\ \text{mA} - 21\ \text{mA})}{21\ \text{mA}}$$

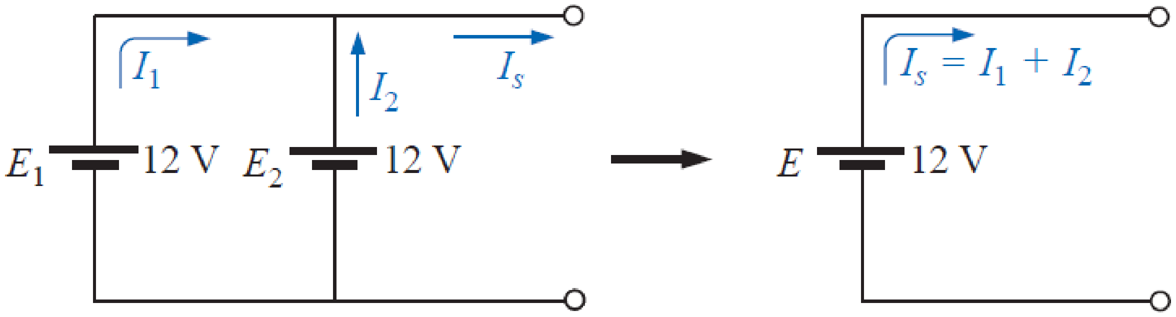
$$= 7\ \Omega\left(\frac{6}{21}\right) = \frac{42\ \Omega}{21} = \mathbf{2\ \Omega}$$

An alternative approach is

$$I_2 = I - I_1 \quad \text{(Kirchhoff's current law)}$$
$$= 27\ \text{mA} - 21\ \text{mA} = 6\ \text{mA}$$
$$V_2 = I_2 R_2 = (6\ \text{mA})(7\ \Omega) = 42\ \text{mV}$$
$$V_1 = I_1 R_1 = V_2 = 42\ \text{mV}$$

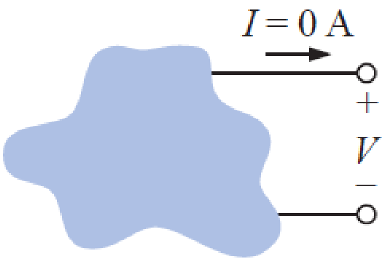
and
$$R_1 = \frac{V_1}{I_1} = \frac{42\ \text{mV}}{21\ \text{mA}} = \mathbf{2\ \Omega}$$

VOLTAGE SOURCES IN PARALLEL



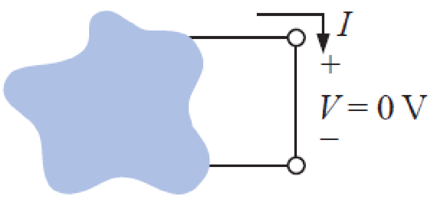
OPEN AND SHORT CIRCUITS

an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.



Open circuit

a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.



Short circuit

Ex.(10) Determine the voltage *Vab* for the network of Fig.(14)

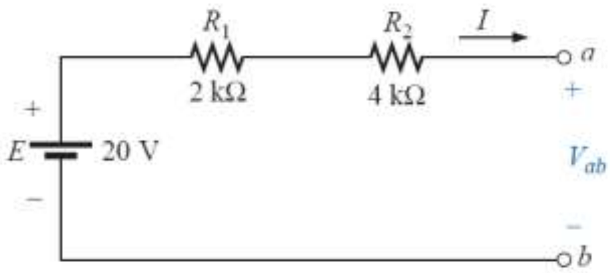


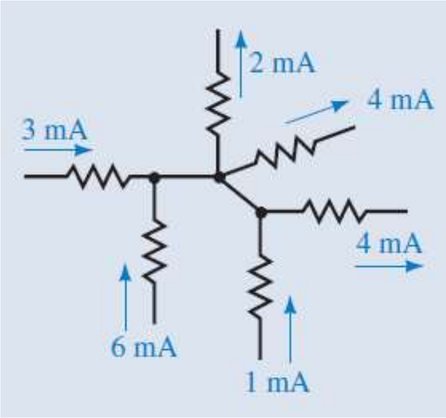
Fig.(14)

Solution: The open circuit requires that I be zero amperes. The voltage drop across both resistors is therefore zero volts since $V = IR = (0)R = 0$ V. Applying Kirchhoff's voltage law around the closed loop,

$$V_{ab} = E = \mathbf{20\ V}$$

Ex.

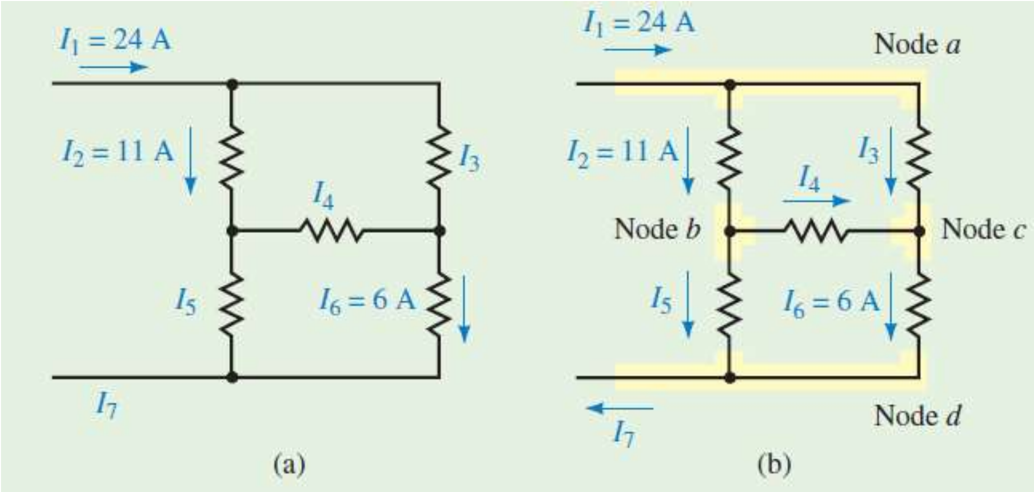
Verify that Kirchhoff's current law applies at the node shown in Figure below:



Answer
3 mA + 6 mA + 1 mA = 2 mA + 4 mA + 4 mA

Ex.

Determine the unknown currents in the network of Figure below



By examining the network, we see that there is only a single source of current $I_1 = 24$ A. Using the analogy of water pipes, we conclude that the current leaving the network is $I_7 = I_1 = 24$ A.

Now, applying Kirchhoff's current law to node *a*, we calculate the current I_3 as follows:

$$I_1 = I_2 + I_3$$

Therefore,

$$I_3 = I_1 - I_2 = 24\text{ A} - 11\text{ A} = 13\text{ A}$$

Similarly, at node *c*, we have

$$I_3 + I_4 = I_6$$

Therefore,

$$I_4 = I_6 - I_3 = 6\text{ A} - 13\text{ A} = -7\text{ A}$$

Although the current I_4 is opposite to the assumed reference direction, we do not change its direction for further calculations. We use the original direction together with the negative sign; otherwise the calculations would be needlessly complicated.

Applying Kirchhoff's current law at node *b*, we get

$$I_2 = I_4 + I_5$$

which gives

$$I_5 = I_2 - I_4 = 11\text{ A} - (-7\text{ A}) = 18\text{ A}$$

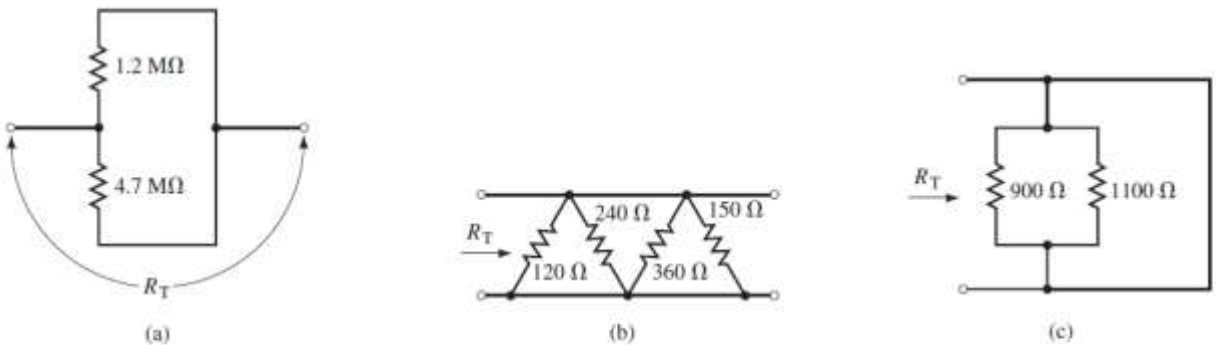
Finally, applying Kirchhoff's current law at node *d* gives

$$I_5 + I_6 = I_7$$

resulting in

$$I_7 = I_5 + I_6 = 18\text{ A} + 6\text{ A} = 24\text{ A}$$

<u>EX</u>. Determine the total resistance of each network of Figure below:



network (a)

$$R_T = \frac{(1.2\ M\Omega)(4.7\ M\Omega)}{1.2\ M\Omega + 4.7\ M\Omega} = 0.956\ M\Omega$$

network (b)

$$\frac{1}{R_T} = \frac{1}{120\ \Omega} + \frac{1}{240\ \Omega} + \frac{1}{360\ \Omega} + \frac{1}{150\ \Omega} = 0.02194\ S$$

$$R_T = \frac{1}{0.02194\ S} = 45.6\ \Omega$$

network (c)

$R_T = 0\ \Omega$ (short circuit)

series-parallel networks :

Series-parallel networks are networks that contain ***both series and parallel circuit configurations.***

Example 1: calculate the source current (I_S) ; (I_B);(I_C) for the cct. of fig(1):

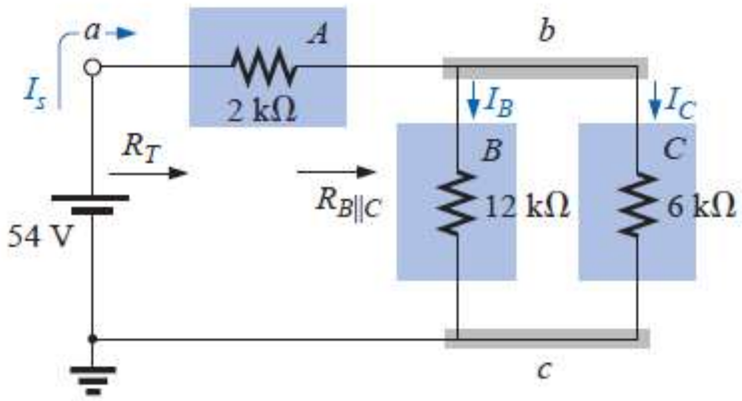


Fig.(1)

Sol:

$$R_{B\|C} = R_B \,||\, R_C = \frac{(12\ \text{k}\Omega)(6\ \text{k}\Omega)}{12\ \text{k}\Omega + 6\ \text{k}\Omega} = 4\ \text{k}\Omega$$

$$\begin{aligned} R_T &= R_A + R_{B\|C} \\ &= 2\ \text{k}\Omega + 4\ \text{k}\Omega = \mathbf{6\ k\Omega} \end{aligned}$$

The result is an equivalent network, as shown in Fig. 2

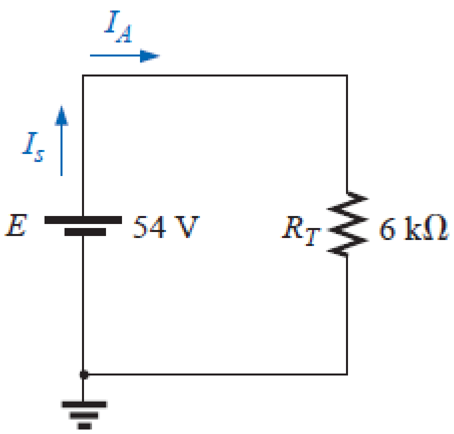


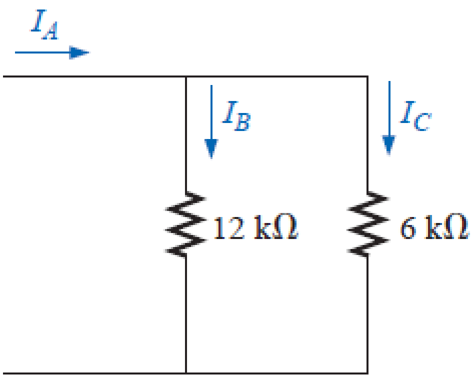
Fig.(2)

$$I_s = \frac{E}{R_T} = \frac{54\text{ V}}{6\text{ k}\Omega} = \mathbf{9\ mA}$$

and, since the source and R_A are in series,

$$I_A = I_s = 9\text{ mA}$$

We can then use the equivalent network of Fig.(3) to determine I_B and I_C using the current divider rule:



Fig(3)

$$I_B = \frac{6\text{ k}\Omega(I_s)}{6\text{ k}\Omega + 12\text{ k}\Omega} = \frac{6}{18}I_s = \frac{1}{3}(9\text{ mA}) = \mathbf{3\ mA}$$

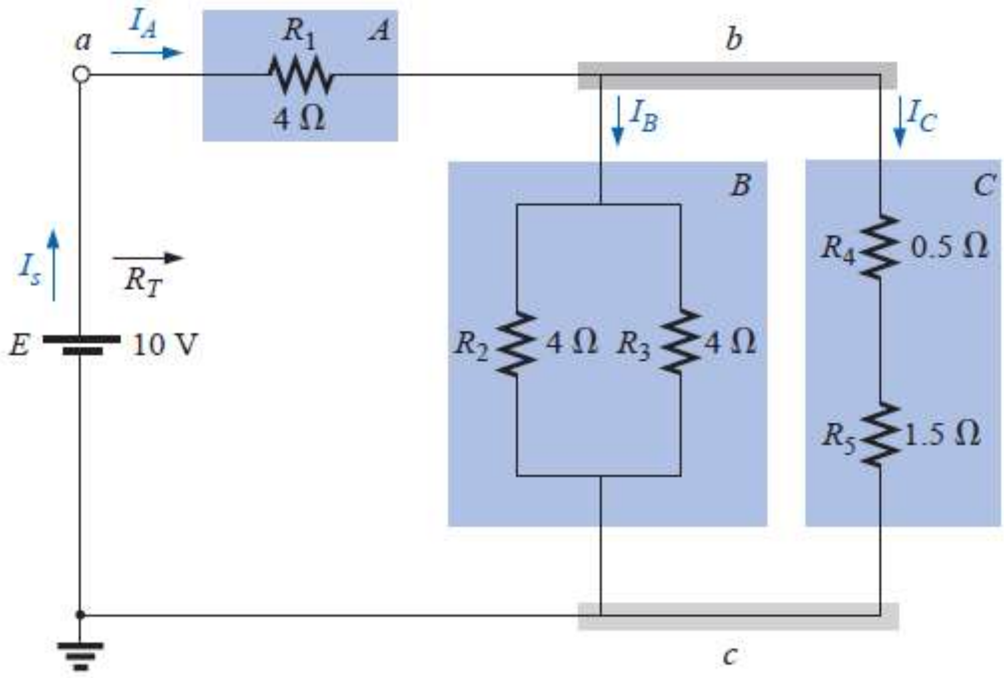
$$I_C = \frac{12\text{ k}\Omega(I_s)}{12\text{ k}\Omega + 6\text{ k}\Omega} = \frac{12}{18}I_s = \frac{2}{3}(9\text{ mA}) = \mathbf{6\ mA}$$

or, applying Kirchhoff's current law,

$$I_C = I_s - I_B = 9\text{ mA} - 3\text{ mA} = \mathbf{6\ mA}$$

<u>Example 2:</u> for the cct. of fig (4) calculate:

R_A , R_B , R_C , R_T , I_T , I_A , I_B , I_C , I_{R1} , I_{R2} , I_{R3} , I_{R4} , I_{R5}



Fig(4)

Sol. : By re-drawing the cct. of fig.(4) we get an equivalent cct.of fig(5):

$$A: \quad R_A = 4\ \Omega$$

$$B: \quad R_B = R_2 \parallel R_3 = R_{2\parallel 3} = \frac{R}{N} = \frac{4\ \Omega}{2} = 2\ \Omega$$

$$C: \quad R_C = R_4 + R_5 = R_{4,5} = 0.5\ \Omega + 1.5\ \Omega = 2\ \Omega$$

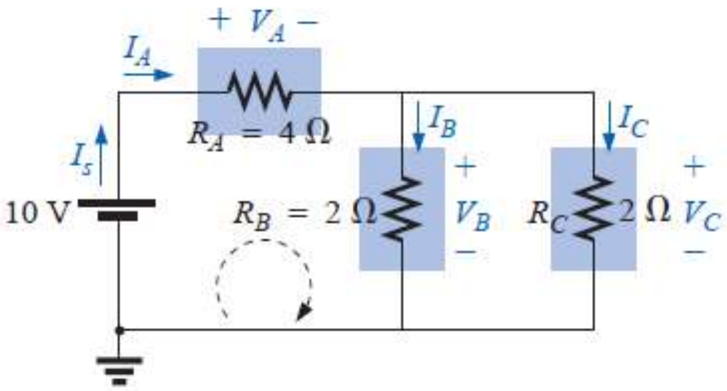


Fig (5)

Blocks B and C are still in parallel, and

$$R_{B\|C} = \frac{R}{N} = \frac{2\ \Omega}{2} = 1\ \Omega$$

$$R_T = R_A + R_{B\|C}$$
$$= 4\ \Omega + 1\ \Omega = \mathbf{5\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{10\text{ V}}{5\ \Omega} = \mathbf{2\text{ A}}$$

$$I_A = I_s = \mathbf{2\text{ A}}$$

$$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\text{ A}}{2} = \mathbf{1\text{ A}}$$

Returning to the network of Fig. 4, we have:

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = \mathbf{0.5\text{ A}}$$

The voltages V_A, V_B, and V_C from either figure are

$$V_A = I_A R_A = (2\text{ A})(4\ \Omega) = \mathbf{8\text{ V}}$$
$$V_B = I_B R_B = (1\text{ A})(2\ \Omega) = \mathbf{2\text{ V}}$$
$$V_C = V_B = \mathbf{2\text{ V}}$$

Applying Kirchhoff's voltage law for the loop indicated in Fig. (5), we obtain:

$$\Sigma_{\circlearrowright} V = E - V_A - V_B = 0$$
$$E = V_A + V_B = 8\text{ V} + 2\text{ V}$$
$$10\text{ V} = 10\text{ V} \quad \text{(checks)}$$

Example 3 : for the circuit in the fig(6) find:

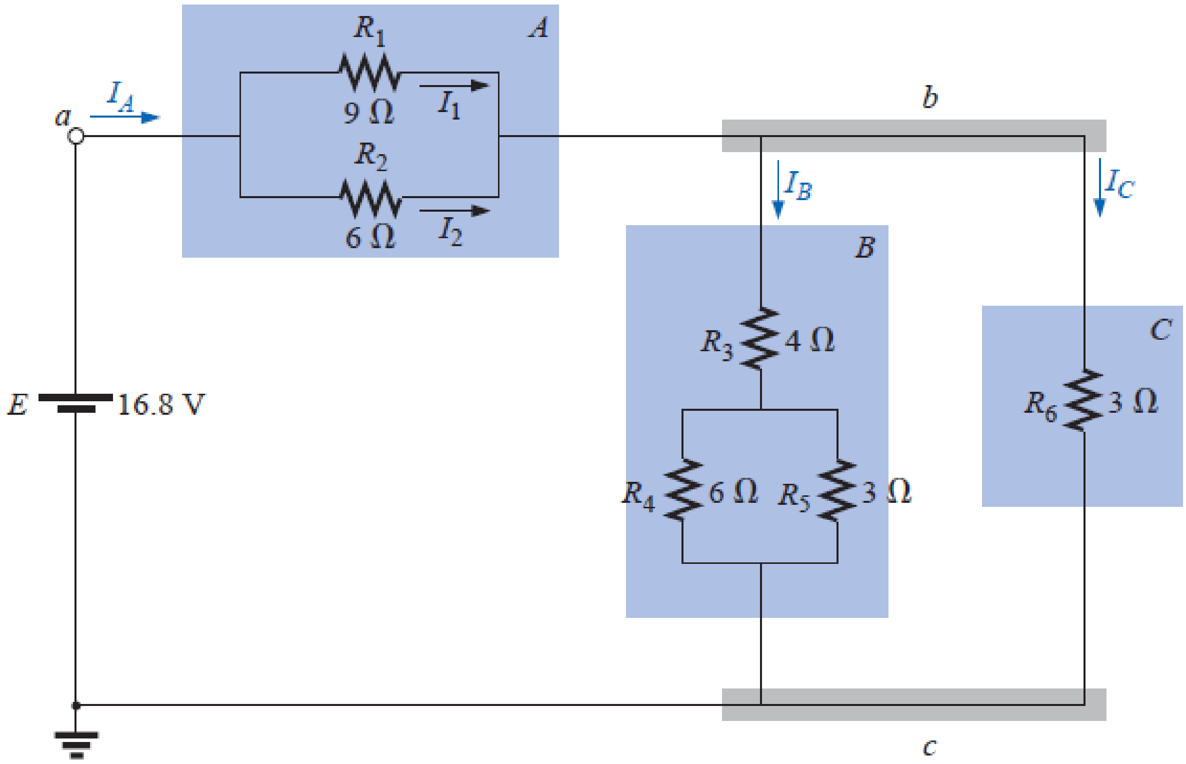
R_T , R_A , R_B , R_C , I_S , I_A , I_B , I_C , I_1 , I_2

SOL.

$$R_A = R_{1\|2} = \frac{(9\ \Omega)(6\ \Omega)}{9\ \Omega + 6\ \Omega} = \frac{54\ \Omega}{15} = 3.6\ \Omega$$

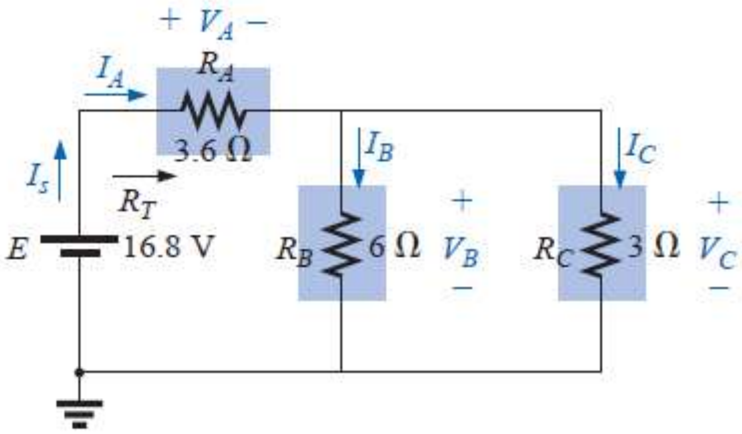
$$R_B = R_3 + R_{4\|5} = 4\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 4\ \Omega + 2\ \Omega = 6\ \Omega$$

$$R_C = 3\ \Omega$$



Fig(6)

The network of Fig.(6) can then be redrawn in reduced form, as shown in Fig. (7).



Fig(7)

$$R_T = R_A + R_{B\|C} = 3.6\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega}$$

$$= 3.6\ \Omega + 2\ \Omega = \mathbf{5.6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{16.8\ \text{V}}{5.6\ \Omega} = \mathbf{3\ A}$$

$$I_A = I_s = \mathbf{3\ A}$$

Applying the current divider rule yields

$$I_B = \frac{R_C I_A}{R_C + R_B} = \frac{(3\ \Omega)(3\ \text{A})}{3\ \Omega + 6\ \Omega} = \frac{9\ \text{A}}{9} = \mathbf{1\ A}$$

By Kirchhoff's current law,

$$I_C = I_A - I_B = 3\ \text{A} - 1\ \text{A} = \mathbf{2\ A}$$

By Ohm's law,

$$V_A = I_A R_A = (3\ \text{A})(3.6\ \Omega) = \mathbf{10.8\ V}$$

$$V_B = I_B R_B = V_C = I_C R_C = (2\ \text{A})(3\ \Omega) = \mathbf{6\ V}$$

Returning to the original network (Fig. 6) and applying the current divider rule:

$$I_1 = \frac{R_2 I_A}{R_2 + R_1} = \frac{(6\ \Omega)(3\ \text{A})}{6\ \Omega + 9\ \Omega} = \frac{18\ \text{A}}{15} = \mathbf{1.2\ A}$$

By Kirchhoff's current law,

$$I_2 = I_A - I_1 = 3\ \text{A} - 1.2\ \text{A} = \mathbf{1.8\ A}$$

EXAMPLE 4: Find the indicated currents and voltages for the network of Fig.(8)

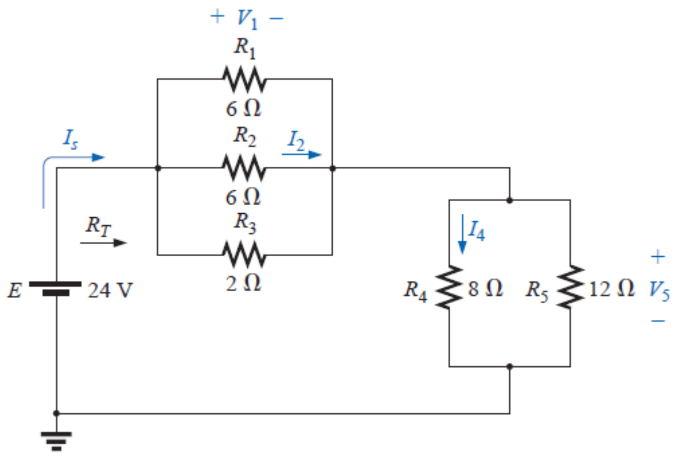
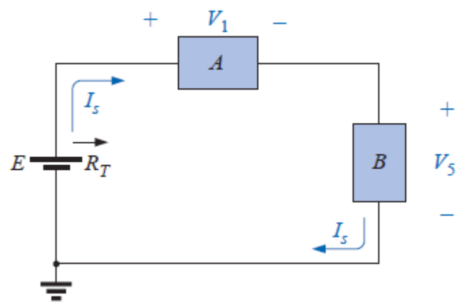


Fig.(8)



fig(9)

Block diagram for Fig. 8

$$R_{1\|2} = \frac{R}{N} = \frac{6\ \Omega}{2} = 3\ \Omega$$

$$R_A = R_{1\|2\|3} = \frac{(3\ \Omega)(2\ \Omega)}{3\ \Omega + 2\ \Omega} = \frac{6\ \Omega}{5} = 1.2\ \Omega$$

$$R_B = R_{4\|5} = \frac{(8\ \Omega)(12\ \Omega)}{8\ \Omega + 12\ \Omega} = \frac{96\ \Omega}{20} = 4.8\ \Omega$$

The reduced form of Fig.(8)will then appear as shown in Fig.(9) :

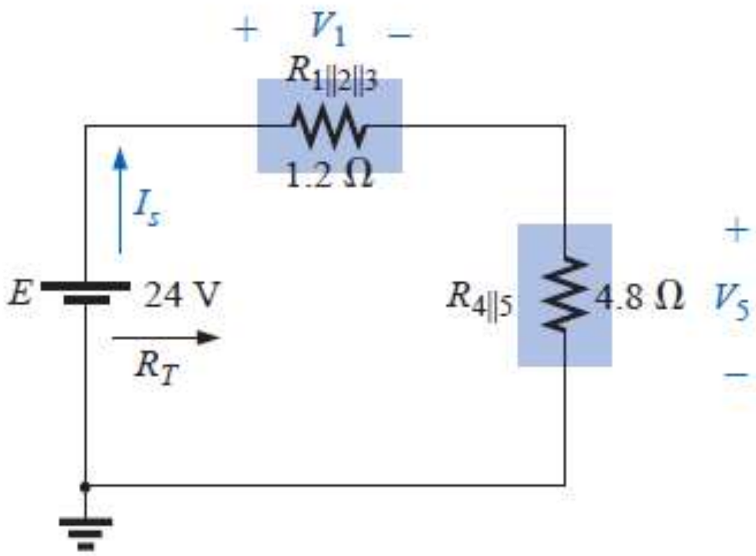


Fig.(9)

$$R_T = R_{1\|2\|3} + R_{4\|5} = 1.2\ \Omega + 4.8\ \Omega = \mathbf{6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{24\text{ V}}{6\ \Omega} = \mathbf{4\ A}$$

with

$$V_1 = I_s R_{1\|2\|3} = (4\text{ A})(1.2\ \Omega) = \mathbf{4.8\ V}$$

$$V_5 = I_s R_{4\|5} = (4\text{ A})(4.8\ \Omega) = \mathbf{19.2\ V}$$

Applying Ohm's law,

$$I_4 = \frac{V_5}{R_4} = \frac{19.2\text{ V}}{8\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8\text{ V}}{6\ \Omega} = \mathbf{0.8\ A}$$

Example 5 :

a. Find the voltages V_1, V_3, and V_{ab} for the network of Fig. (10).
b. Calculate the source current I_S.

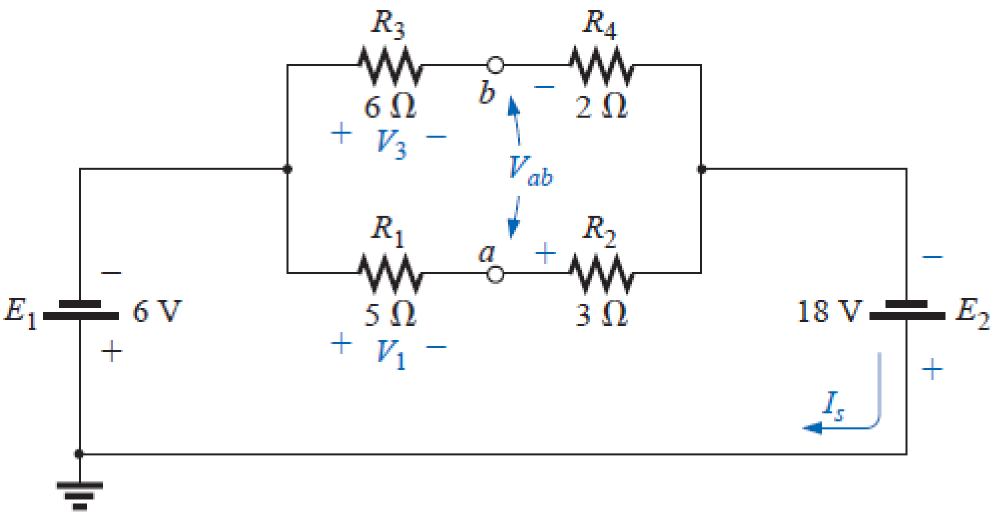


Fig.(10)

The cct. is re-drawn as in fig(11):

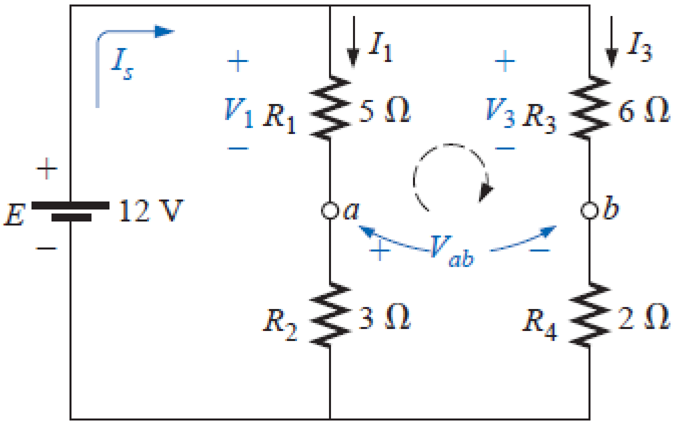


Fig.(11)

a.

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(5\ \Omega)(12\ \text{V})}{5\ \Omega + 3\ \Omega} = \frac{60\ \text{V}}{8} = \mathbf{7.5\ V}$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{(6\ \Omega)(12\ \text{V})}{6\ \Omega + 2\ \Omega} = \frac{72\ \text{V}}{8} = \mathbf{9\ V}$$

The open-circuit voltage *Vab* is determined by applying Kirchhoff's voltage law around the indicated loop of Fig.(11) in the clockwise direction starting at terminal *a*.

$$+V_1 - V_3 + V_{ab} = 0$$

and $$V_{ab} = V_3 - V_1 = 9\ \text{V} - 7.5\ \text{V} = \mathbf{1.5\ V}$$

b. By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5\ \text{V}}{5\ \Omega} = 1.5\ \text{A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{9\ \text{V}}{6\ \Omega} = 1.5\ \text{A}$$

Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\ \text{A} + 1.5\ \text{A} = \mathbf{3\ A}$$

EXAMPLE 6: For the network of Fig. (12), determine the voltages *V*1 and *V*2 and the current *I*.

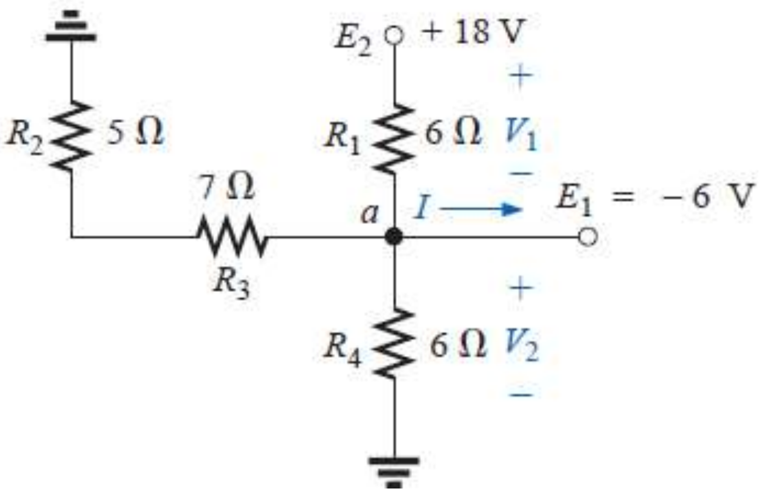


Fig.(12)

The cct. is re-drawn as in fig.(13)

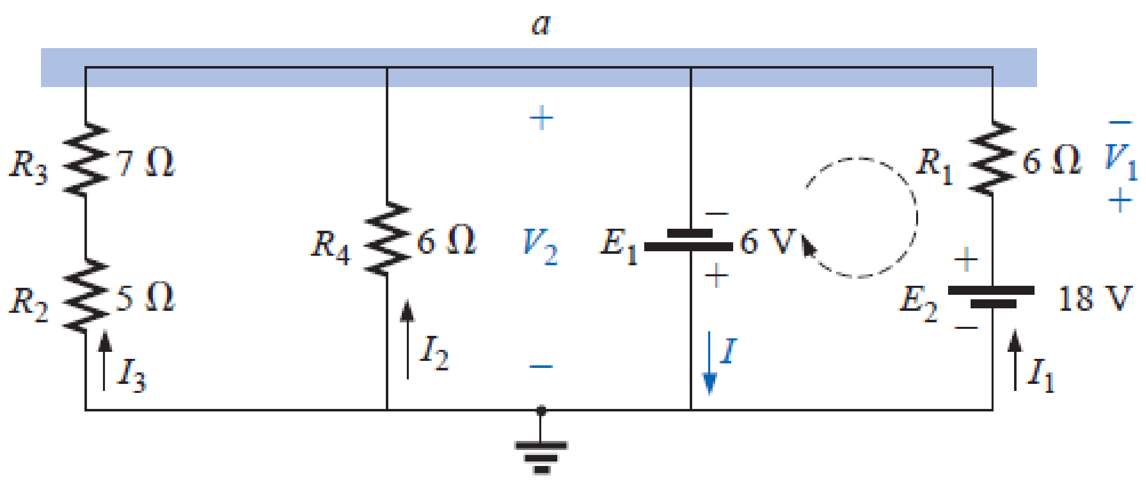


Fig.(13)

$$V_2 = -E_1 = \mathbf{-6\ V}$$

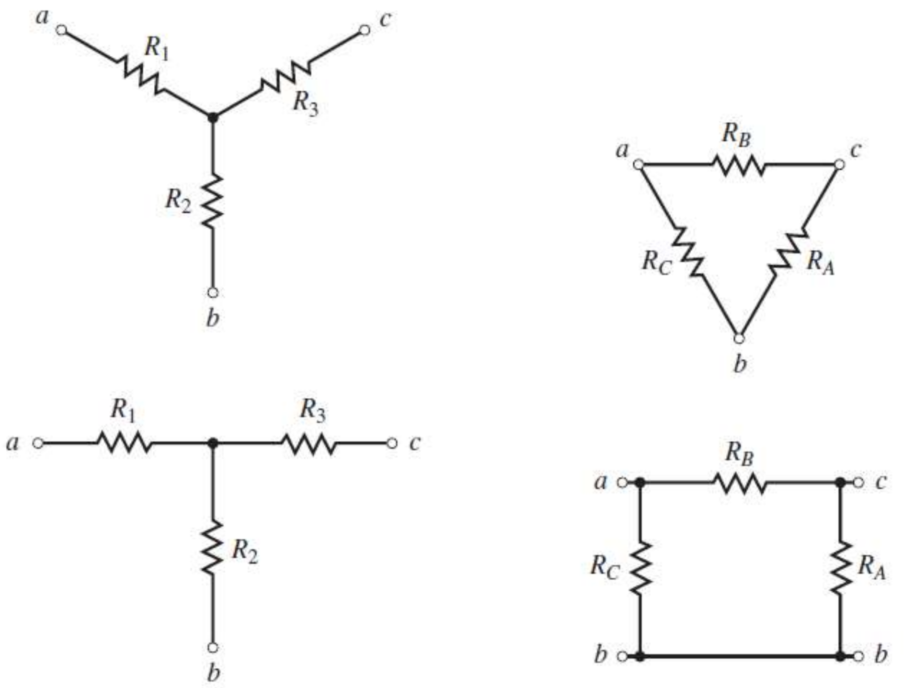
$$-E_1 + V_1 - E_2 = 0$$

and $$V_1 = E_2 + E_1 = 18\text{ V} + 6\text{ V} = \mathbf{24\text{ V}}$$

Applying Kirchhoff's current law to node *a* yields

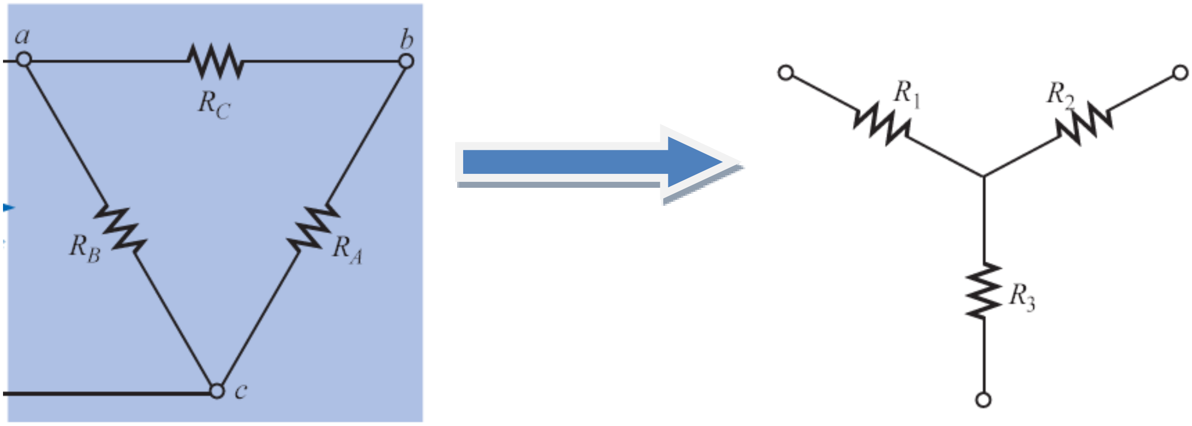
$$\begin{aligned}
I &= I_1 + I_2 + I_3 \\
&= \frac{V_1}{R_1} + \frac{E_1}{R_4} + \frac{E_1}{R_2 + R_3} \\
&= \frac{24\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{6\ \Omega} + \frac{6\text{ V}}{12\ \Omega} \\
&= 4\text{ A} + 1\text{ A} + 0.5\text{ A} \\
I &= \mathbf{5.5\text{ A}}
\end{aligned}$$

Wye (Y ⅄) or Tee (┬ ┴) network	**Delta (∇ Δ) or Pi (╨ ╥) network**



(a) Wye (Y) or Tee (T) network (b) Delta (Δ) or Pi (Π) network

1- Convert **∇ (R_A, R_B, R_C) to** Y **(R_1, R_2, R_3)**



$$R_1 = \frac{RB \times RC}{RA + RB + RC}$$

$$R2 = \frac{RA \times RC}{RA + RB + RC}$$

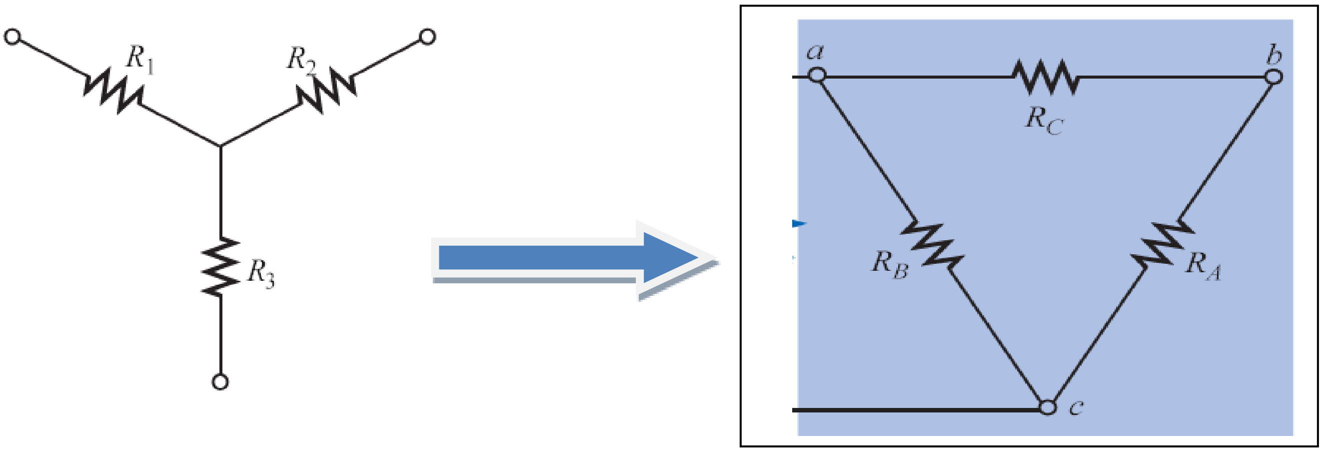
$$R3 = \frac{RA \times RB}{RA + RB + RC}$$

☺ Note :

If $R_A = R_B = R_C$,

$$R_Y = \frac{R_\Delta}{3}$$

2- convert Y **(R_1, R_2, R_3)** to Δ **(R_A, R_B, R_C)**



$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

☺ **Note:**

if $R_1=R_2=R_3$

$$R_\Delta = 3R_Y$$

Example (1):

Convert the (Δ) of fig (1) to (Y):

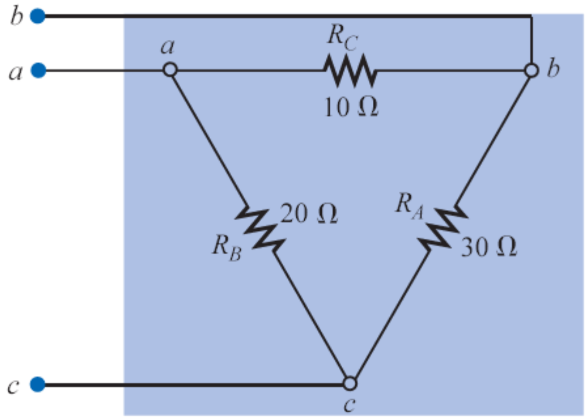


Fig.(1)

Solution:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = \mathbf{3\tfrac{1}{3}\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

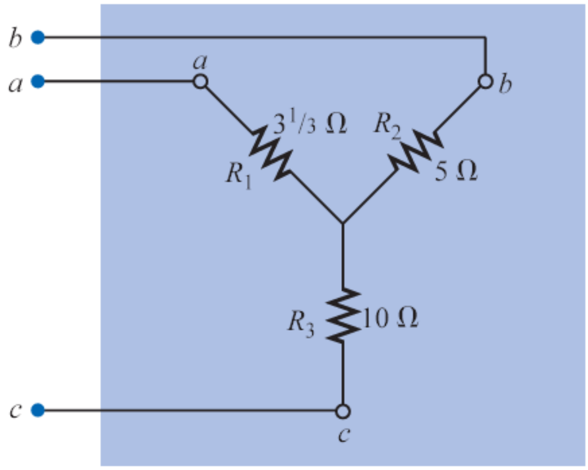
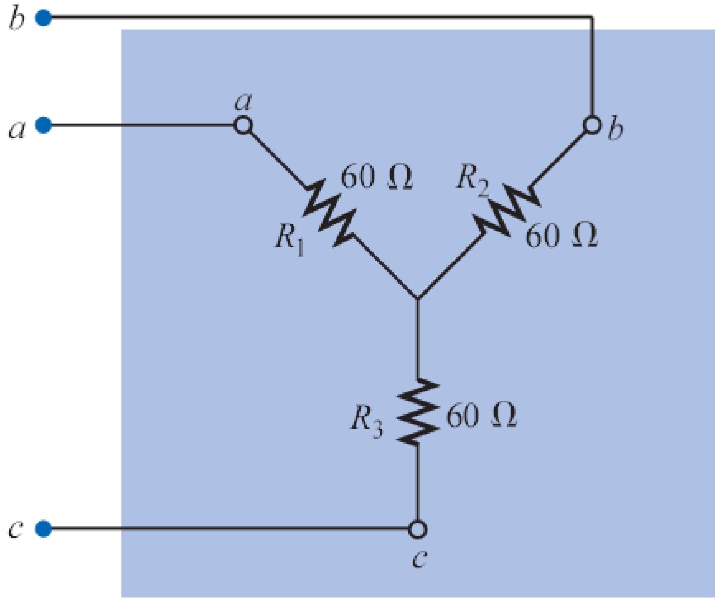


Fig.(1-a) Y equivalent to Δ

Example(2) : Convert the Y of Fig. (2) to a Δ.



Fig(2)

Solution:

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$= \frac{(60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega) + (60\ \Omega)(60\ \Omega)}{60\ \Omega}$$

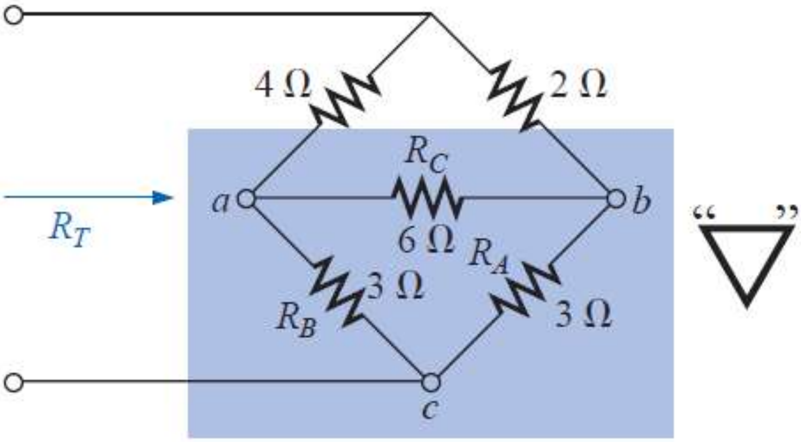
$$= \frac{3600\ \Omega + 3600\ \Omega + 3600\ \Omega}{60} = \frac{10{,}800\ \Omega}{60}$$

$$R_A = \mathbf{180\ \Omega}$$

$$R_\Delta = 3R_Y = 3(60\ \Omega) = 180\ \Omega$$

$$R_B = R_C = \mathbf{180\ \Omega}$$

Example(3): Find the total resistance of the network of Fig. (3), where *RA* = 3 Ω , *RB* = 3 Ω, and *RC* = 6Ω



Fig(3)

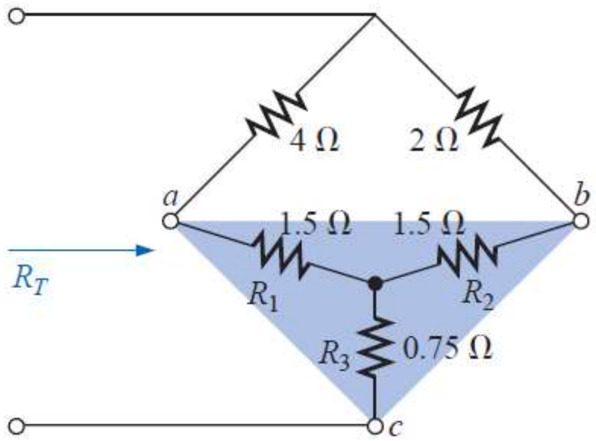
Solution:

Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$



Fig(3-a)(Y connection for Δ of shaded area of fig.(3)

2Ω+1.5Ω=3.5Ω

4Ω+1.5Ω=5.5Ω

3.5 // 5.5 =2.138Ω

R_T=2.138+0.75=2.88 Ω

Example(4): Find the total resistance of the network of Fig.(4)

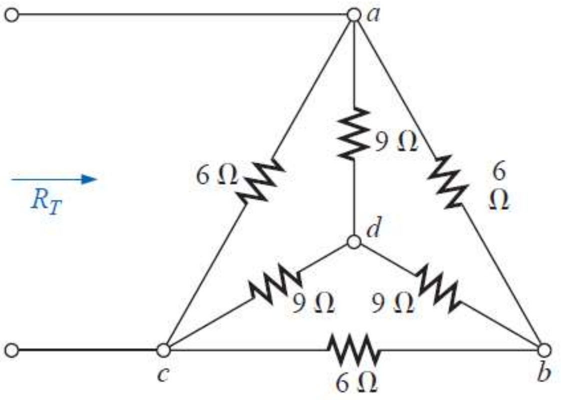
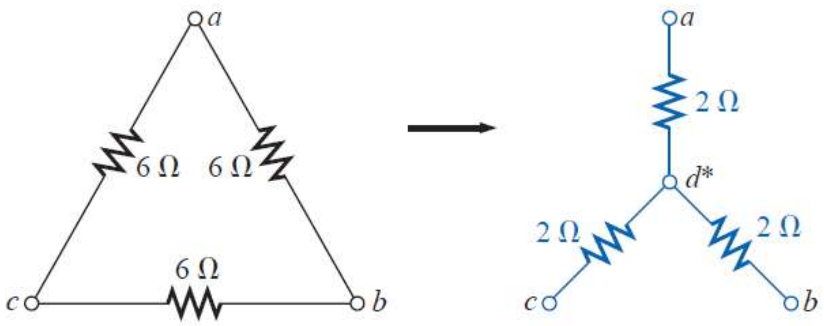


Fig.(4)

$$R_Y = \frac{R_\Delta}{3} = \frac{6\ \Omega}{3} = 2\ \Omega$$



fig(4-a)

Converting the Δ configuration of Fig. (14) to a Y configuration.

The network then appears as shown in Fig. (4-b).

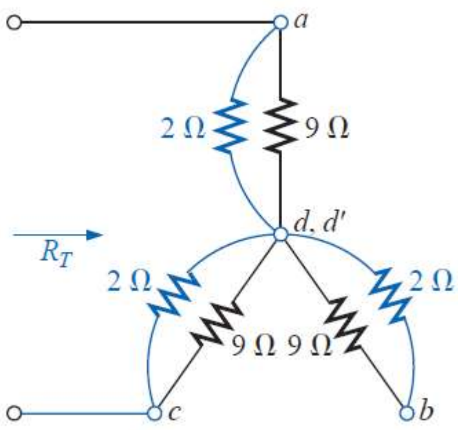
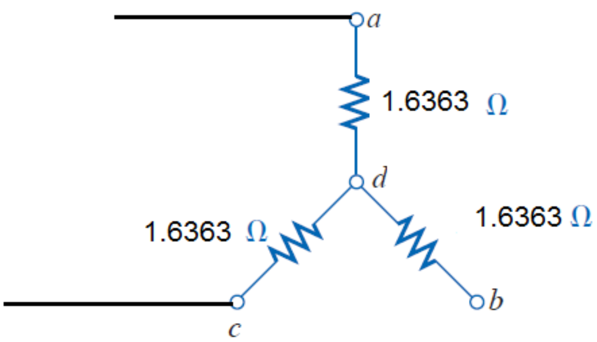


Fig.(4-b)

Substituting the (Y) configuration for the converted (Δ)
into the network of Fig. (4)

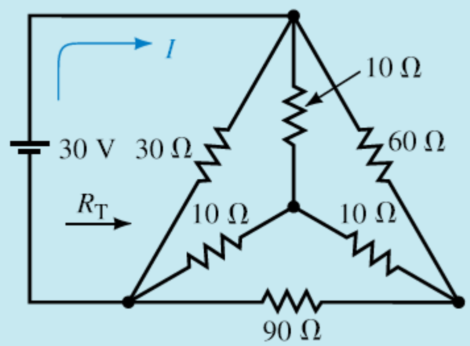
$$RT = \left(\frac{2*9}{2+9}\right) = 1.6363$$



RT = 1.6363 + 1.6363 = 3.2727 Ω

Ex. Given the circuit of Figure below find the total resistance ,RT, and the total current(I) .

Sol:



$$R_\Delta = 3(10\ \Omega) = 30\ \Omega$$

The resulting circuit is shown in Figure 8–46(a).

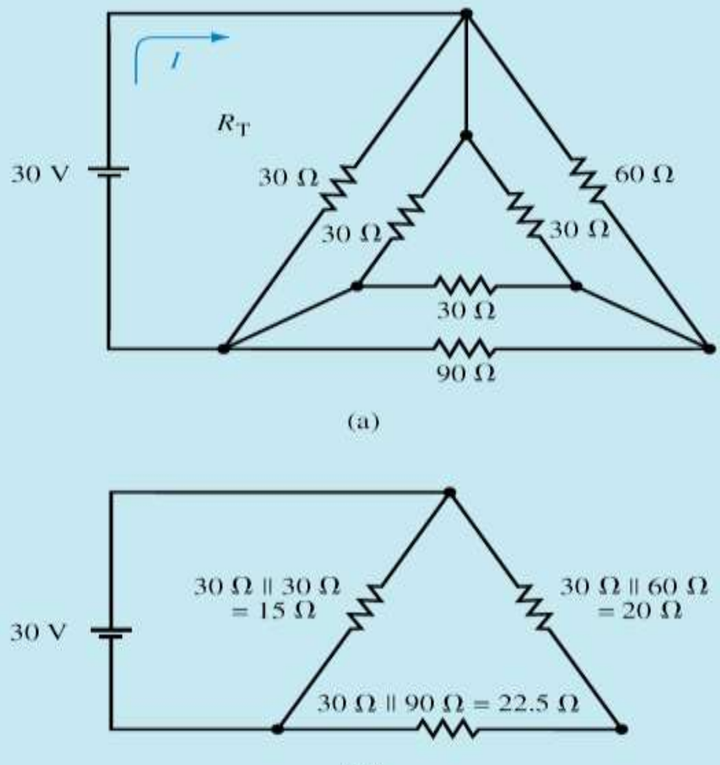


FIGURE 8–46

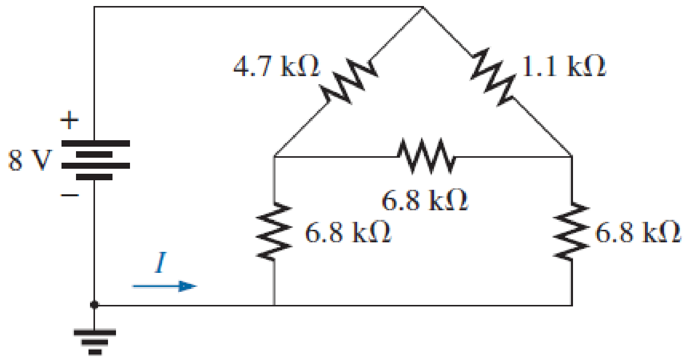
We see that the sides of the resulting "Δ" are in parallel, which allows us to simplify the circuit even further as shown in Figure 8–46(b). The total estance of the circuit is now easily determined as

$$R_T = 15\ \Omega \| (20\ \Omega + 22.5\ \Omega)$$
$$= 11.09\ \Omega$$

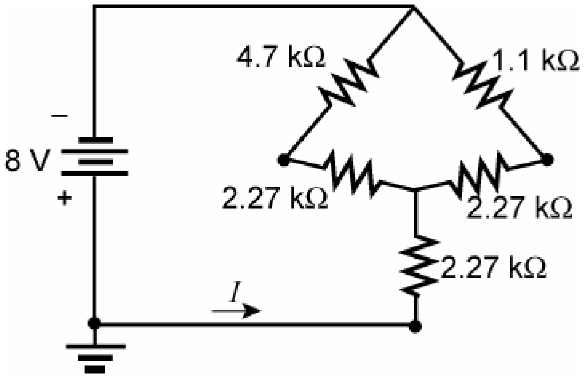
This results in a circuit current of

$$I = \frac{30\ V}{11.09\ \Omega} = 2.706\ A$$

Ex. Using a Δ-Y or Y-Δ conversion, find the current *I* in the network in Fig. below:



Sol.



(Complete the solution !)………(اكمل الحل !)

Exercise 1: find the total current (I_T) for the circuit shown in fig.(1):

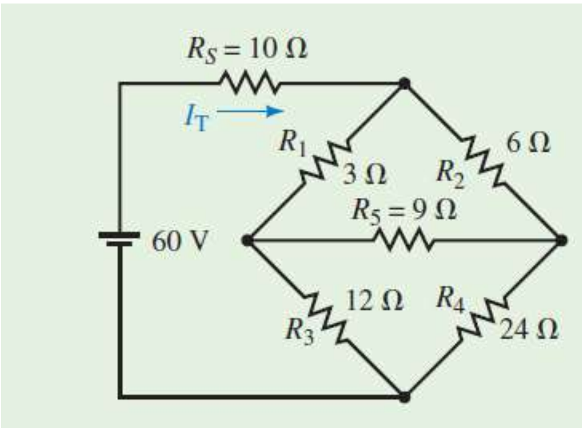
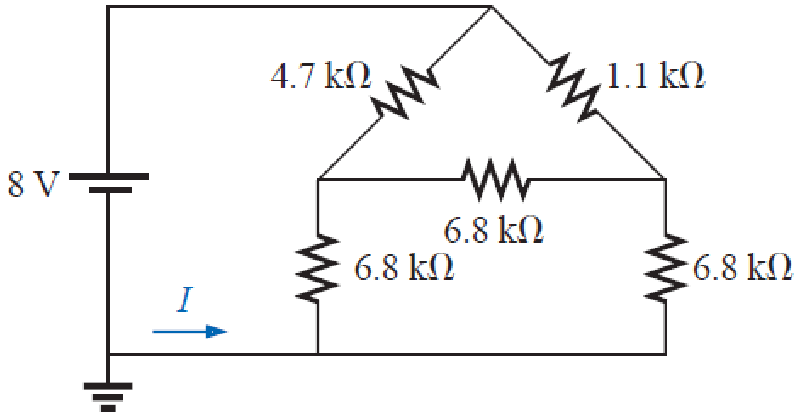


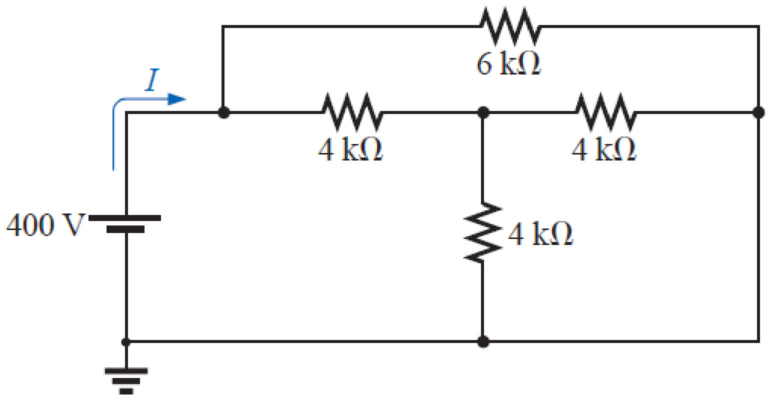
Fig.(1)

Exercise (2) Using a Δ-Y or Y-Δ conversion, find the current I in the network of Fig. (2)



Fig(2)

Exercise 3 :Determine the current (I) for the network of Fig.(3):



Fig(3)

2) NODAL ANALYSIS (FORMAT APPROACH)

1. Choose a reference node and assign a subscripted voltage label to the (N - 1) remaining nodes of the network.
2. The number of equations required for a complete solution is equal to the number of subscripted voltages (N-1). Column 1 of each equation is formed by summing the conductance's tied to the node of interest and multiplying the result by that subscripted nodal voltage.
3. We must now consider the mutual terms that, as noted in the preceding example, are always subtracted from the first column. It is possible to have more than one mutual term if the nodal voltage of current interest has an element in common with more than one other nodal voltage. This will be demonstrated in an example to follow. Each mutual term is the product of the mutual conductance and the other nodal voltage tied to that onductance.
4. The column to the right of the equality sign is the algebraic sum of the current sources tied to the node of interest. A current source is assigned a positive sign if it supplies current to a node and a negative sign if it raws current from the node.
5. Solve the resulting simultaneous equations for the desired voltages.

Ex. Write the nodal equations for the network of Fig (1).

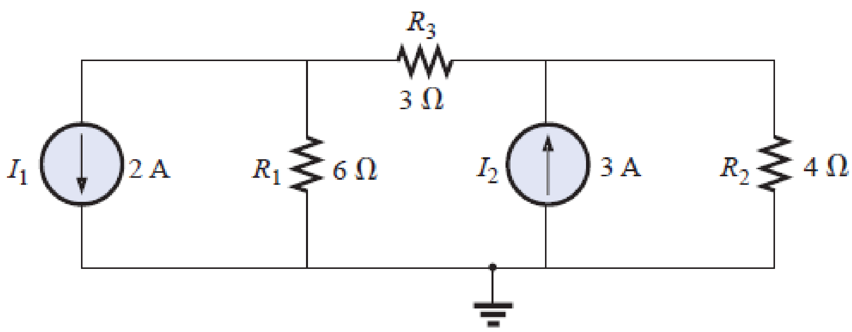


Fig.(1)

Sol.

Step 1: **The figure is redrawn with assigned subscripted voltages in Fig.(2)**

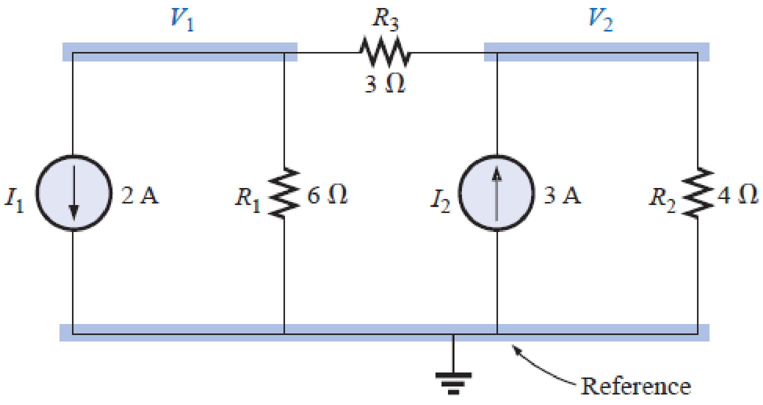


Fig.(2)

Steps 2 to 4:

Drawing current from node 1

$$V_1: \underbrace{\left(\frac{1}{6\,\Omega} + \frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 1}}} V_1 - \underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_2 = -2\text{ A}$$

Supplying current to node 2

$$V_2: \underbrace{\left(\frac{1}{4\,\Omega} + \frac{1}{3\,\Omega}\right)}_{\substack{\text{Sum of}\\ \text{conductances}\\ \text{connected}\\ \text{to node 2}}} V_2 - \underbrace{\left(\frac{1}{3\,\Omega}\right)}_{\substack{\text{Mutual}\\ \text{conductance}}} V_1 = +3\text{ A}$$

and

$$\frac{1}{2}V_1 - \frac{1}{3}V_2 = -2$$

$$-\frac{1}{3}V_1 + \frac{7}{12}V_2 = 3$$

Ex. Find the voltage across the 3Ω resistor of Fig. (3) by nodal analysis

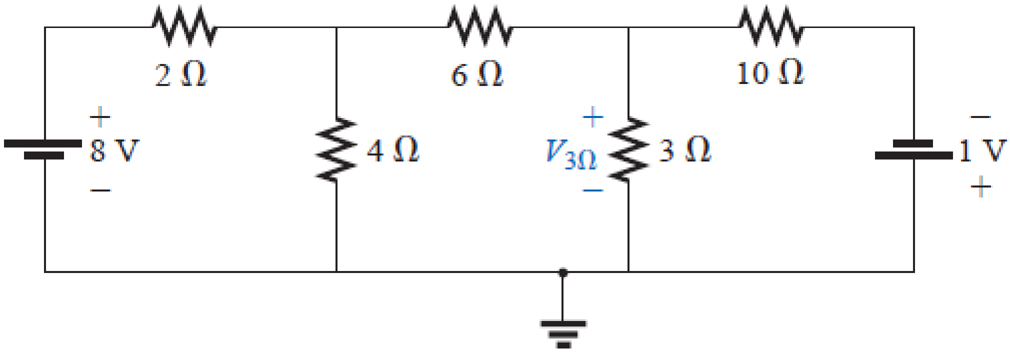


Fig.(3)

Solution: Converting voltage source to current source and choosing nodes:

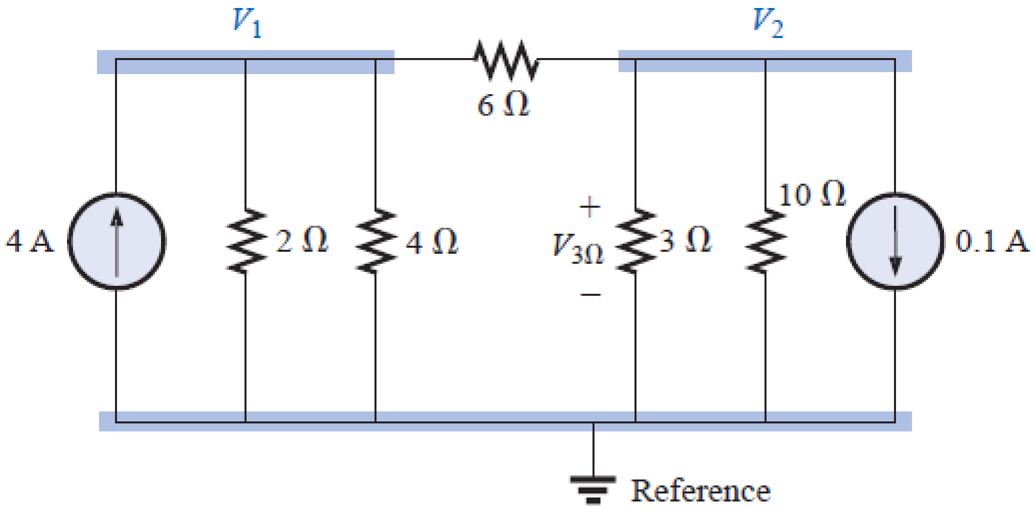


Fig.(4)

$$\left.\begin{aligned}\left(\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}\right)V_1-\left(\frac{1}{6\,\Omega}\right)V_2 &= +4\text{ A}\\ \left(\frac{1}{10\,\Omega}+\frac{1}{3\,\Omega}+\frac{1}{6\,\Omega}\right)V_2-\left(\frac{1}{6\,\Omega}\right)V_1 &= -0.1\text{ A}\end{aligned}\right\}$$

$$\frac{11}{12}V_1 - \frac{1}{6}V_2 = 4$$
$$-\frac{1}{6}V_1 + \frac{3}{5}V_2 = -0.1$$

resulting in

$$11V_1 - 2V_2 = +48$$
$$-5V_1 + 18V_2 = -3$$

and

$$V_2 = V_{3\Omega} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{-33 + 240}{198 - 10} = \frac{207}{188} = \mathbf{1.101\ V}$$

Ex. Using nodal analysis, determine the potential across the 4Ω resistor in Fig.(5):

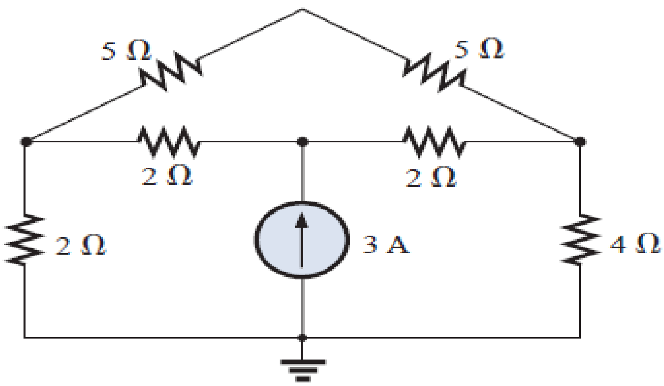
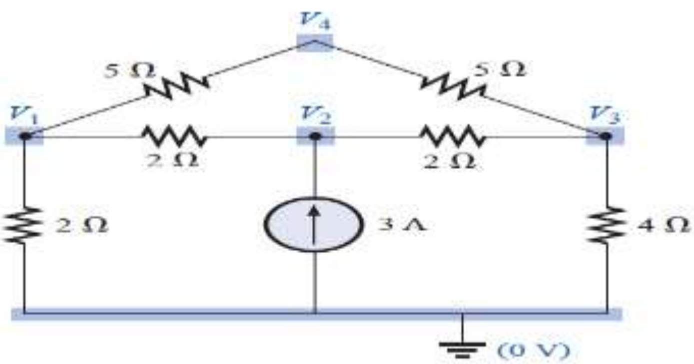


Fig.(5)



Fig,(6)

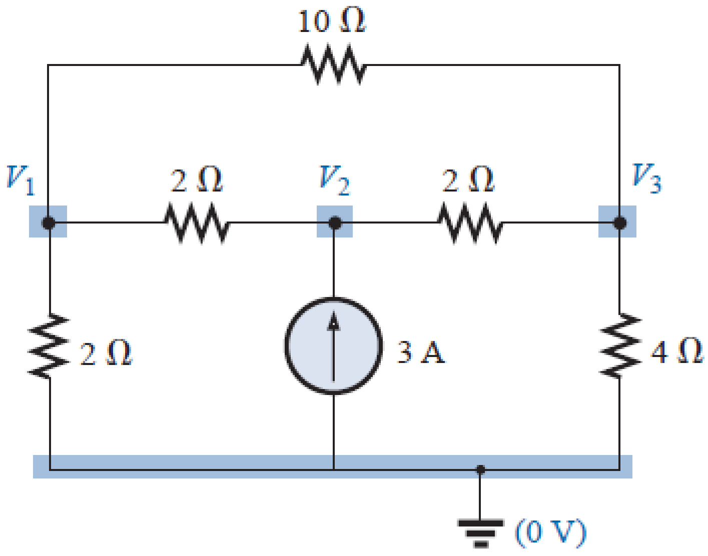


Fig.(7)

$$V_1:\quad \left(\frac{1}{2\,\Omega}+\frac{1}{2\,\Omega}+\frac{1}{10\,\Omega}\right)V_1-\left(\frac{1}{2\,\Omega}\right)V_2-\left(\frac{1}{10\,\Omega}\right)V_3=0$$

$$V_2:\quad \left(\frac{1}{2\,\Omega}+\frac{1}{2\,\Omega}\right)V_2-\left(\frac{1}{2\,\Omega}\right)V_1-\left(\frac{1}{2\,\Omega}\right)V_3=3\text{ A}$$

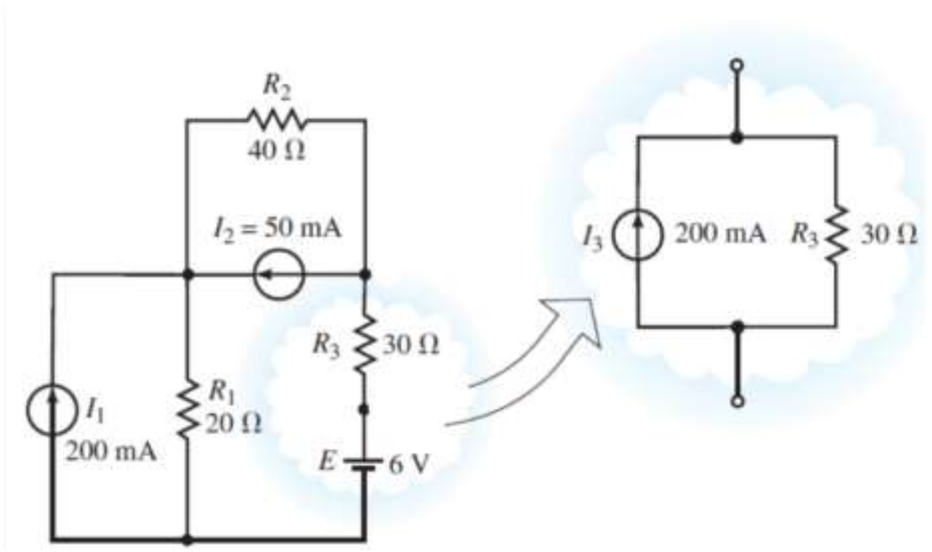
$$V_3:\quad \left(\frac{1}{10\,\Omega}+\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}\right)V_3-\left(\frac{1}{2\,\Omega}\right)V_2-\left(\frac{1}{10\,\Omega}\right)V_1=0$$

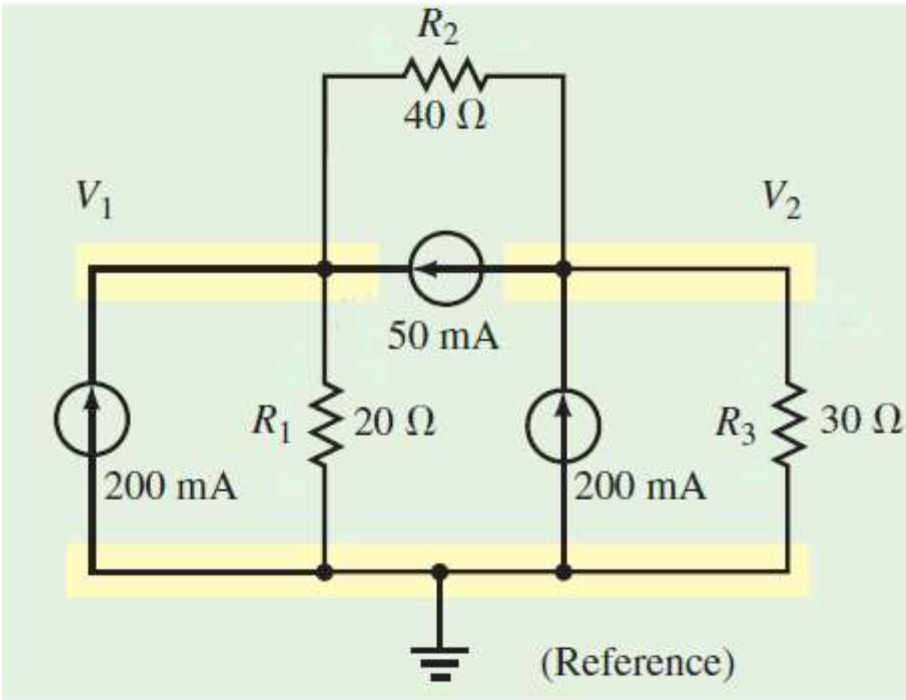
$$\begin{aligned}1.1V_1-0.5V_2-0.1V_3&=0\\ V_2-0.5V_1-0.5V_3&=3\\ 0.85V_3-0.5V_2-0.1V_1&=0\end{aligned}$$

$$V_3=V_{4\Omega}=\frac{\begin{vmatrix}1.1 & -0.5 & 0\\ -0.5 & +1 & 3\\ -0.1 & -0.5 & 0\end{vmatrix}}{\begin{vmatrix}1.1 & -0.5 & -0.1\\ -0.5 & +1 & -0.5\\ -0.1 & -0.5 & +0.85\end{vmatrix}}=\mathbf{4.645\ V}$$

Ex.

Write the nodal equations for the network of Fig. below :



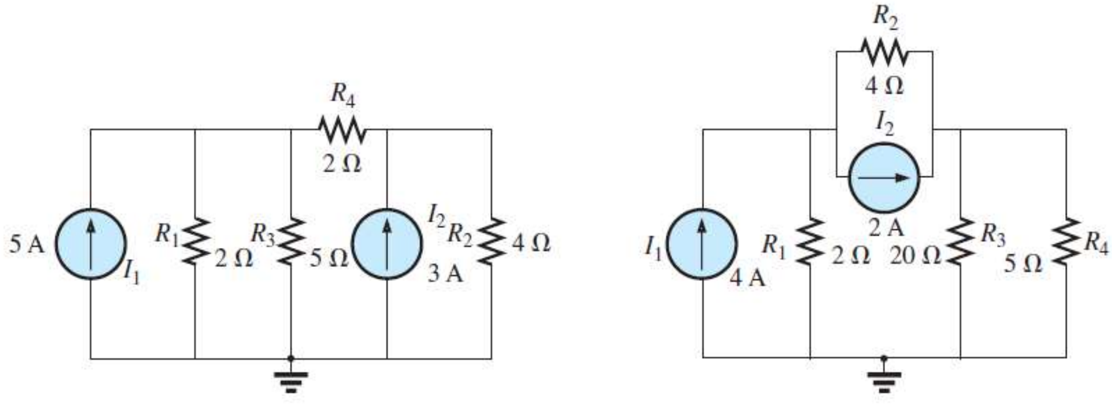


$$\left(\frac{1}{20\ \Omega}+\frac{1}{40\ \Omega}\right)V_1-\left(\frac{1}{40\ \Omega}\right)V_2 = 200\text{ mA}+ 50\text{ mA}$$

$$-\left(\frac{1}{40\ \Omega}\right)V_1+\left(\frac{1}{30\ \Omega}+\frac{1}{40\ \Omega}\right)V_2 = 200\text{ mA} - 50\text{ mA}$$

Complete the solution………

Exercise Using the format approach, write the nodal equations for the networks in Fig. below. Using determinants, solve for the nodal voltages

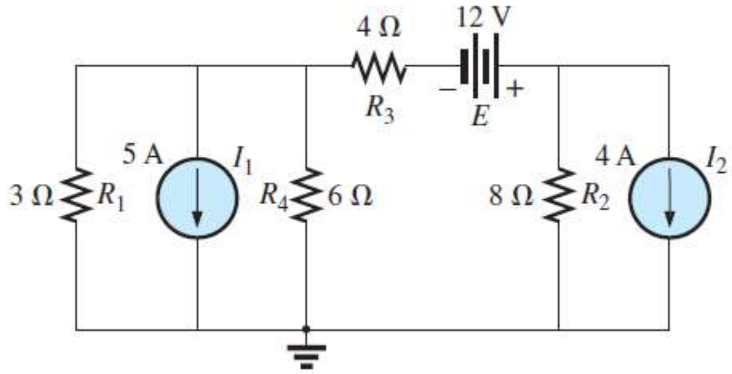


(a) (b)

Exercise a. Write the nodal equations for the networks in Fig. below

b. Using determinants, solve for the nodal voltages.

c. Determine the magnitude and polarity of the voltage across each resistor



1-SUPERPOSITION THEOREM :

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

$$\text{Number of networks to be analyzed} = \text{Number of independent sources}$$

طريقة الحل

تطبق هذه الطريقة عند وجود مصدرين أو اكثر ويجب ان نبقي مصدر واحد فقط عند الحل

1- **Remove a voltage source when applying this theorem**
2- **The difference in potential between the terminals of the voltage source must be set to zero (short circuit).**
3- **Removing a current source requires that its terminals be opened (open circuit).**
4- **Any internal resistance or conductance associated with the displaced sources is not eliminated but must still be considered.**

Figure (A) reviews the various substitutions required when removing an ideal source, and Figure (B) reviews the substitutions with practical sources that have an internal resistance.

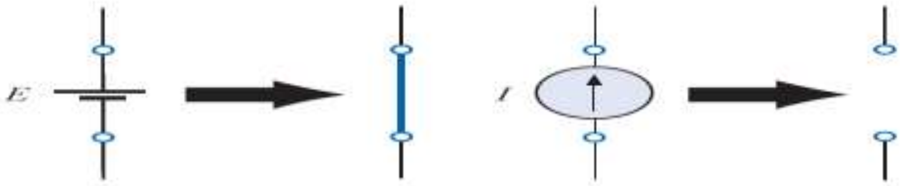


Fig. (A) Removing the effects of ideal sources.

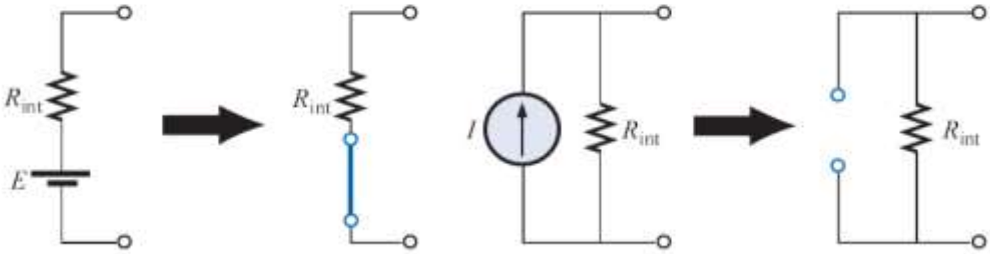


Fig.(B) Removing the effects of practical sources

Note the total power delivered to a resistive element must be determined using the total current through or the total voltage across the element and cannot be determined by a simple sum of the power levels established by each source.

1) **We must determine the value and direction of current in each circuit and denote the current I` ,I``,I``` and so on.**
2) **then we determine the resultant of the currents depending on its directions.**

EXAMPLE (1): Determine (I1) for the network of Fig. (1).

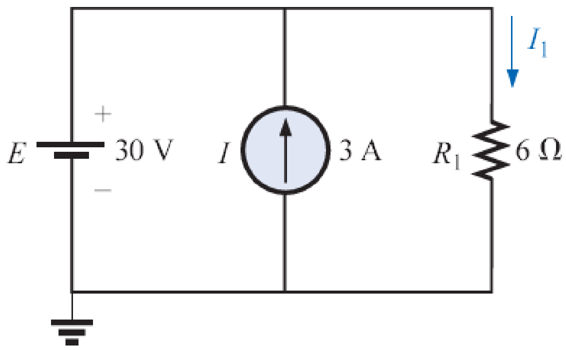


Fig.(1)

Solution:

1-Setting E = 0 V for the network of Fig. (1) results in the network of Fig. 1(a), where a short-circuit equivalent has replaced the 30-V source.

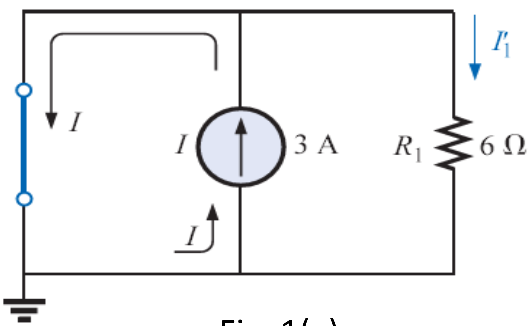


Fig. 1(a)

As shown in Fig.1(a), the source current will choose the short circuit path, and $I'_1 = 0$ A. If we applied the current divider rule,

$$I'_1 = \frac{R_{sc} I}{R_{sc} + R_1} = \frac{(0\ \Omega)I}{0\ \Omega + 6\ \Omega} = 0\ \text{A}$$

2- Setting (I)to zero amperes will result in the network of Fig1(b), with the current Source replaced by an open circuit. Applying Ohm's law,

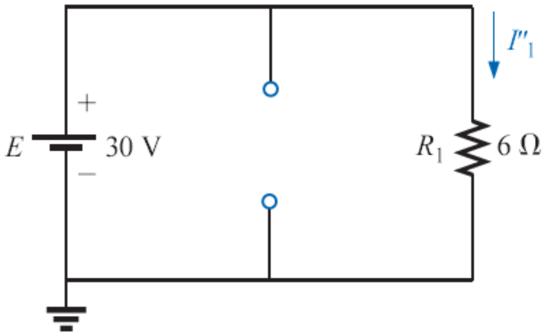


Fig.1(b)

$$I''_1 = \frac{E}{R_1} = \frac{30\ \text{V}}{6\ \Omega} = 5\ \text{A}$$

Since I'_1 and I''_1 have the same defined direction in Fig. 1(a) and (b), the current I_1 is the sum of the two, and:

$$I_1 = I'_1 + I''_1 = 0\ \text{A} + 5\ \text{A} = \mathbf{5\ A}$$

<u>EXAMPLE(2)</u>. Using superposition, determine the current through the 4**Ω** resistor of Fig.(2).

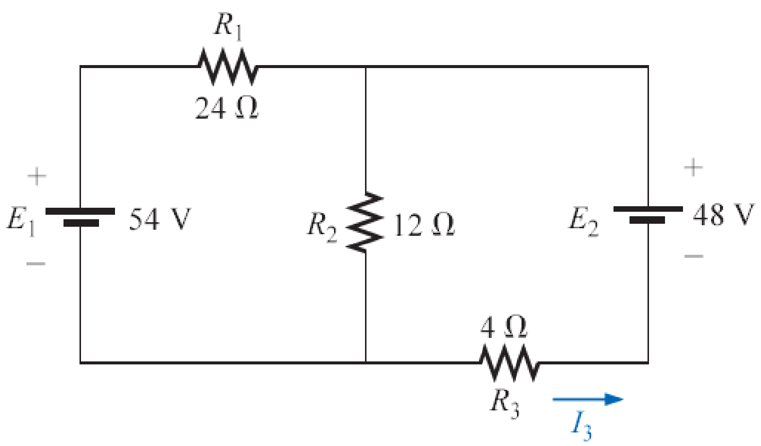


Fig.(2)

1-Considering the effects of a 54V source (Fig.2a)

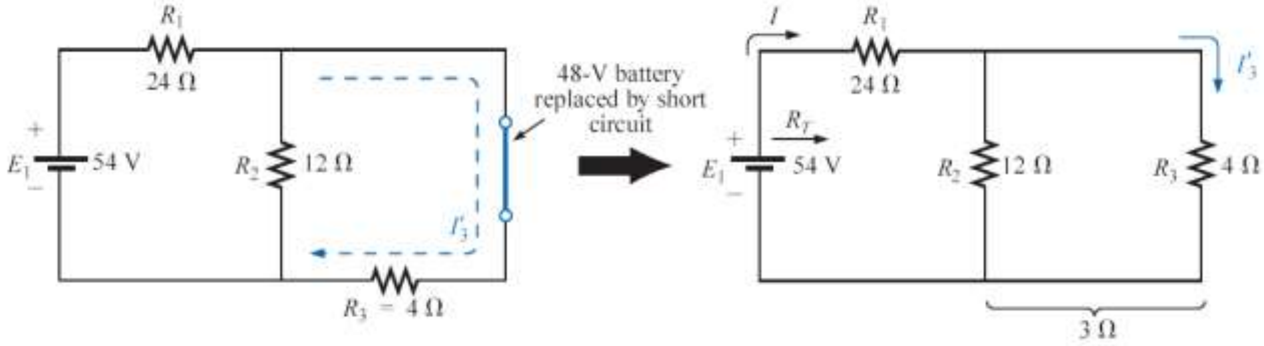


Fig.2(a) The effect of E_1on the current I_3.

Rt= R1+(R2//R3)

Rt=24+(12//4)

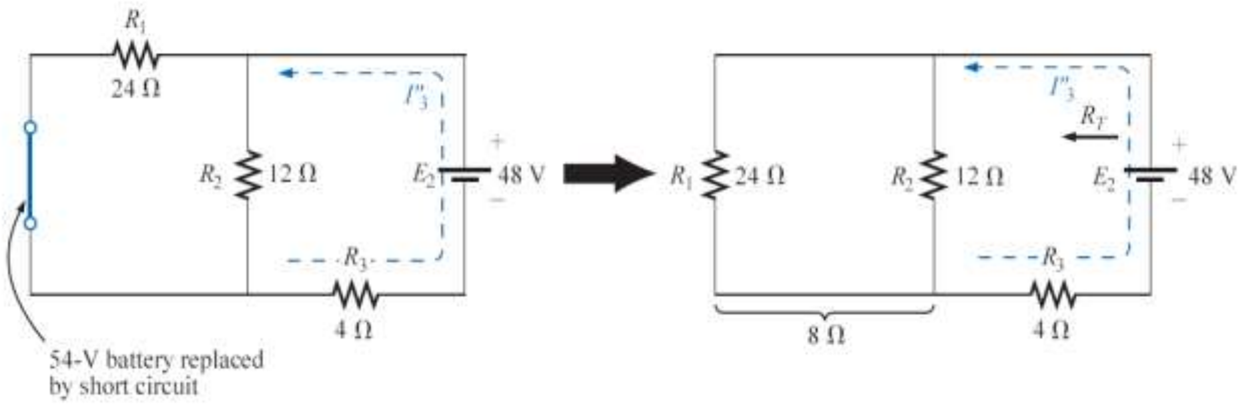
Rt= 27

I (total) = $\frac{E}{Rt} = \frac{54}{27} = 2$ A

Using the current divider rule,

$$I'_3 = \frac{R_2 I}{R_2 + R_3} = \frac{(12\ \Omega)(2\ \text{A})}{12\ \Omega + 4\ \Omega} = \frac{24\ \text{A}}{16} = 1.5\ \text{A}$$

2-Considering the effects of the 48V source (Fig2b):

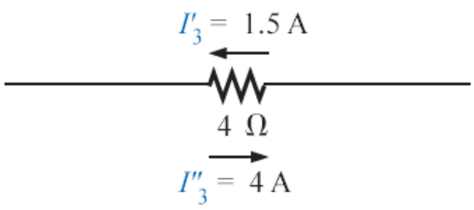


Fig(2b) The effect of E_2on the current I_3.

$$R_T = R_3 + R_1 \parallel R_2 = 4\ \Omega + 24\ \Omega \parallel 12\ \Omega = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

$$I''_3 = \frac{E_2}{R_T} = \frac{48\ \text{V}}{12\ \Omega} = 4\ \text{A}$$

The total current through the 4**Ω** resistor (Fig. 2c) is:



Fig(2c) The resultant current for I_3.

$$I_3 = I''_3 - I'_3 = 4\ \text{A} - 1.5\ \text{A} = 2.5\ \text{A} \qquad (\text{direction of } I''_3)$$

EXAMPLE (9-5) مثال في الكتاب Find the current through the 2Ω resistor of the network of Fig. (9-19). the presence of three sources will result in three different networks to be analyzed.

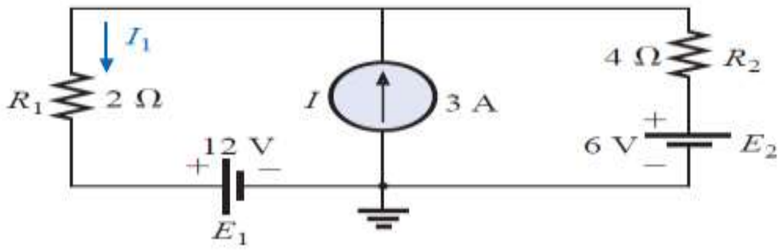


Fig.(9-19)

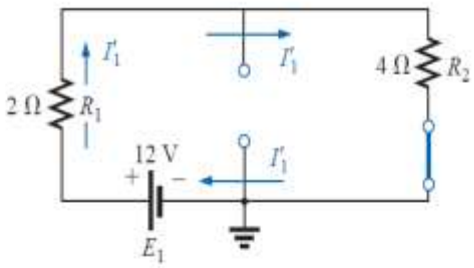


FIG. 9.20
The effect of E_1 on the current I.

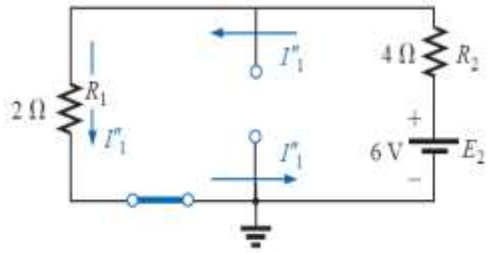


FIG. 9.21
The effect of E_2 on the current I_1.

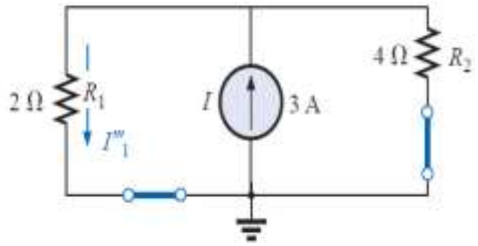


FIG. 9.22
The effect of I on the current I_1.

Solution: *Considering the effect of the 12-V source (Fig. 9.20):*

$$I'_1 = \frac{E_1}{R_1 + R_2} = \frac{12\text{ V}}{2\,\Omega + 4\,\Omega} = \frac{12\text{ V}}{6\,\Omega} = 2\text{ A}$$

Considering the effect of the 6-V source (Fig. 9.21):

$$I''_1 = \frac{E_2}{R_1 + R_2} = \frac{6\text{ V}}{2\,\Omega + 4\,\Omega} = \frac{6\text{ V}}{6\,\Omega} = 1\text{ A}$$

Considering the effect of the 3-A source (Fig. 9.22):
Applying the current divider rule,

$$I'''_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4\,\Omega)(3\text{ A})}{2\,\Omega + 4\,\Omega} = \frac{12\text{ A}}{6} = 2\text{ A}$$

The total current through the 2-Ω resistor appears in Fig. 9.23, and

Same direction as I_1 in Fig. 9.19 — Opposite direction to I_1 in Fig. 9.19

$$I_1 = \overbrace{I''_1 + I'''_1}^{\text{Same direction as } I_1 \text{ in Fig. 9.19}} - \underbrace{I'_1}_{\text{Opposite direction}}$$

$$= 1\text{A} + 2\text{A} - 2\text{A} = \mathbf{1A}$$

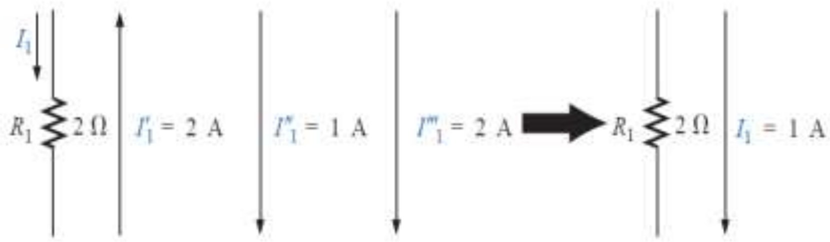
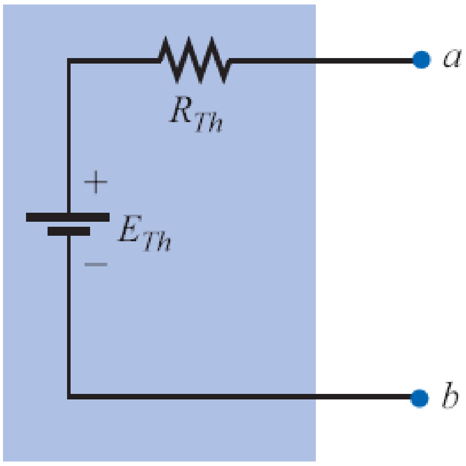


FIG. 9.23
The resultant current I_1.

2-THEVENIN'S THEOREM

Any two-terminal, linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor, as shown in Fig. below.



The following sequence of steps will lead to the proper value of R_{Th} and E_{Th}.

1. Remove that portion of the network across which the Thévenin equivalent circuit is to be found. In Fig. 2(a), this requires that the load resistor R_L be temporarily removed from the network.

2. Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)

R_{Th}:

3. Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)

E_{Th}:

4. Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that will lead to the most confusion and errors. In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.)

Conclusion:

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor R_L between the terminals of the Thévenin equivalent circuit as shown in Fig. 2(b).

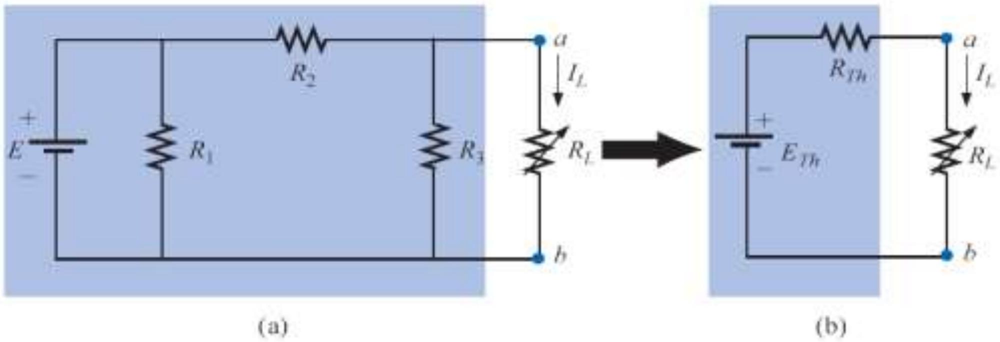
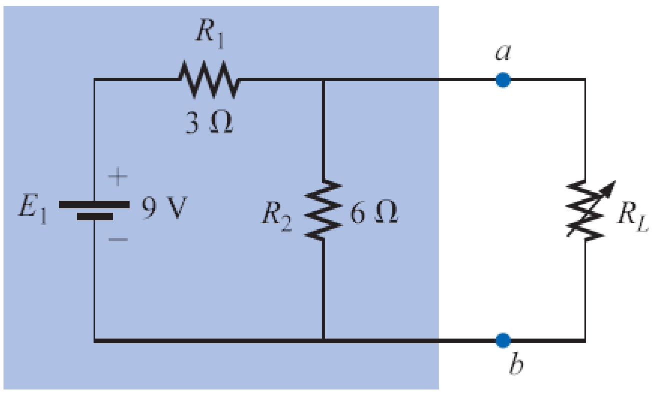


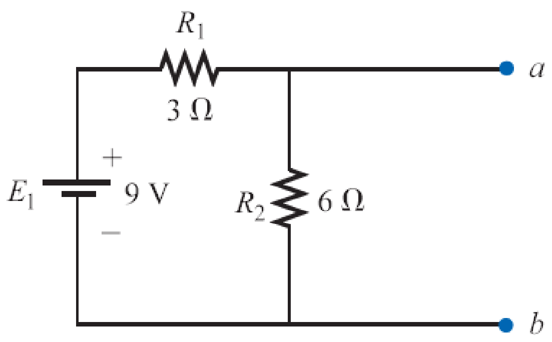
Fig.(a and b)

Substituting the Thévenin equivalent circuit for a complex network.

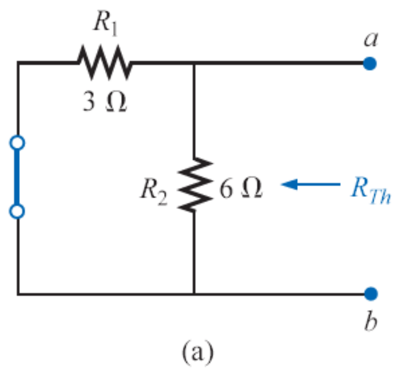
<u>Ex.1</u>: Find the Thévenin equivalent circuit for the network in the <u>shaded</u> area of the network of Fig.(1). Then find the current through R_L for values of 2 **Ω**, 10 **Ω**, and 100 **Ω**.



Fig(1)



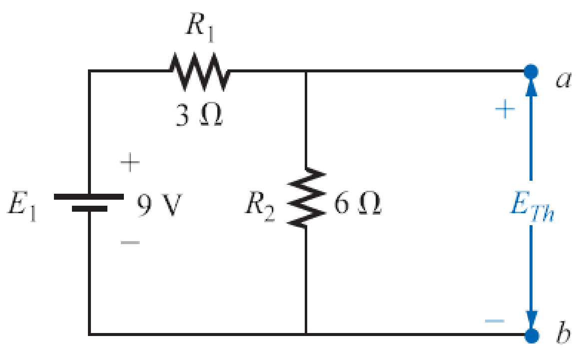
1- Find R_{Th} fig.(1-a)



Fig(1-a) find R_{Th}

$$R_{Th} = R_1 \parallel R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \mathbf{2\ \Omega}$$

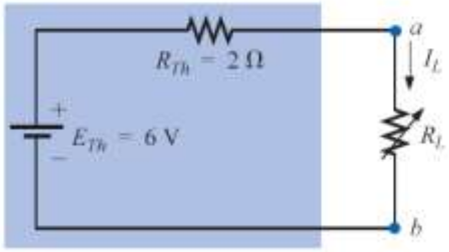
2- Find E_{Th} fig(1-b)



Fig(1-b)

$$E_{Th} = \frac{R_2E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ V)}{6\ \Omega + 3\ \Omega} = \frac{54\ V}{9} = \mathbf{6\ V}$$

3- Substituting the Thévenin equivalent circuit for the network external to R_L fig(1-c)



Fig(1-c)

المطلب الثاني من السؤال

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

$$R_L = 2\ \Omega: \quad I_L = \frac{6\ V}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

$$R_L = 10\ \Omega: \quad I_L = \frac{6\ V}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$$R_L = 100\ \Omega: \quad I_L = \frac{6\ V}{2\ \Omega + 100\ \Omega} = \mathbf{0.059\ A}$$

EXAMPLE 9.10 من الكتاب (Two sources) Find the Thévenin circuit for the network within the shaded area of Fig. (9.49).

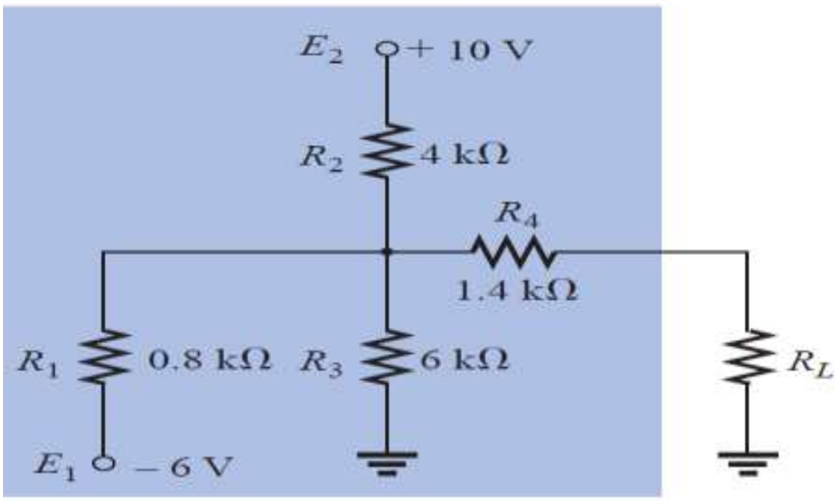


FIG. 9.49
Example 9.10.

Solution: The network is redrawn and *steps 1 and 2* are applied as shown in Fig. 9.50.

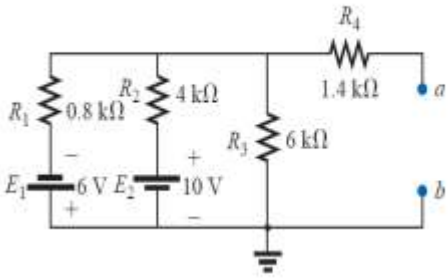


FIG. 9.50
Identifying the terminals of particular interest for the network of Fig. 9.49.

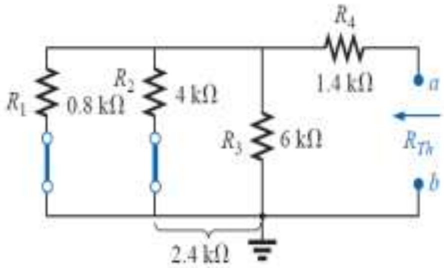


FIG. 9.51
Determining R_{Th} for the network of Fig. 9.50.

Step 3: See Fig. 9.51.

$$\begin{aligned}R_{Th} &= R_4 + R_1 \parallel R_2 \parallel R_3\\ &= 1.4\text{ k}\Omega + 0.8\text{ k}\Omega \parallel 4\text{ k}\Omega \parallel 6\text{ k}\Omega\\ &= 1.4\text{ k}\Omega + 0.8\text{ k}\Omega \parallel 2.4\text{ k}\Omega\\ &= 1.4\text{ k}\Omega + 0.6\text{ k}\Omega\\ &= \mathbf{2\text{ k}\Omega}\end{aligned}$$

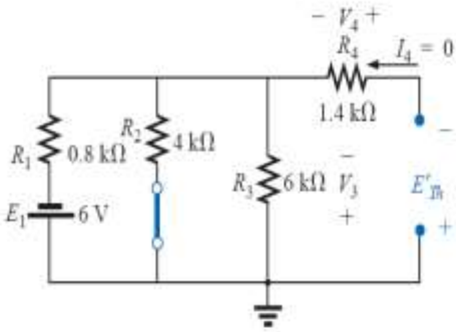


FIG. 9.52
Determining the contribution to E_{Th} from the source E_1 for the network of Fig. 9.50.

Step 4: Applying superposition, we will consider the effects of the voltage source E_1 first. Note Fig. 9.52. The open circuit requires that $V_4 = I_4R_4 = (0)R_4 = 0$ V, and

$$E'_{Th} = V_3$$
$$R'_T = R_2 \parallel R_3 = 4\text{ k}\Omega \parallel 6\text{ k}\Omega = 2.4\text{ k}\Omega$$

Applying the voltage divider rule,

$$V_3 = \frac{R'_T E_1}{R'_T} + R_1 = \frac{(2.4\text{ k}\Omega)(6\text{ V})}{2.4\text{ k}\Omega + 0.8\text{ k}\Omega} = \frac{14.4\text{ V}}{3.2} = 4.5\text{ V}$$
$$E'_{Th} = V_3 = 4.5\text{ V}$$

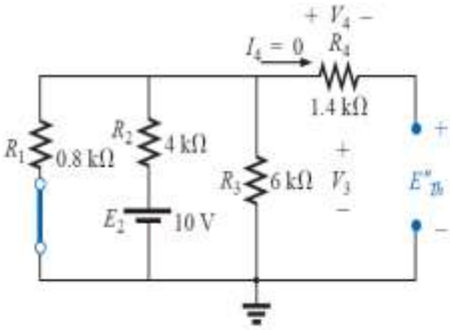


FIG. 9.53
Determining the contribution to E_{Th} from the source E_2 for the network of Fig. 9.50.

For the source E_2, the network of Fig. 9.53 will result. Again, $V_4 = I_4R_4 = (0)R_4 = 0$ V, and

$$E''_{Th} = V_3$$
$$R'_T = R_1 \parallel R_3 = 0.8\text{ k}\Omega \parallel 6\text{ k}\Omega = 0.706\text{ k}\Omega$$

and $$V_3 = \frac{R'_T E_2}{R'_T + R_2} = \frac{(0.706\text{ k}\Omega)(10\text{ V})}{0.706\text{ k}\Omega + 4\text{ k}\Omega} = \frac{7.06\text{ V}}{4.706} = 1.5\text{ V}$$
$$E''_{Th} = V_3 = 1.5\text{ V}$$

Since E'_{Th} and E''_{Th} have opposite polarities,

$$E_{Th} = E'_{Th} - E''_{Th}$$
$$= 4.5\text{ V} - 1.5\text{ V}$$
$$= \mathbf{3\text{ V}} \qquad \text{(polarity of } E'_{Th}\text{)}$$

Step 5: See Fig. 9.54.

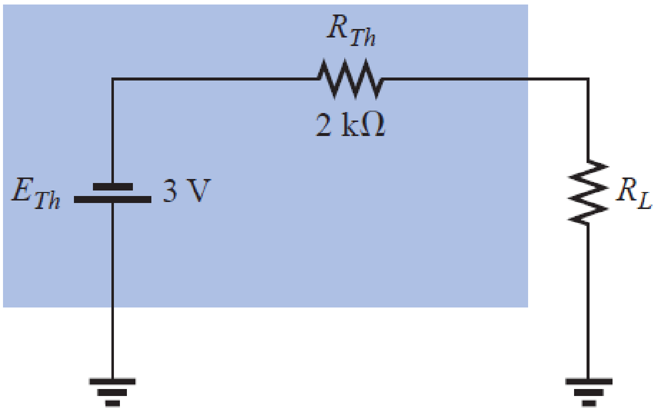
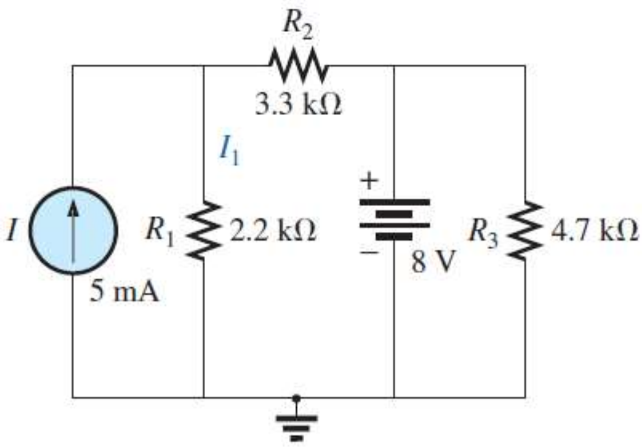
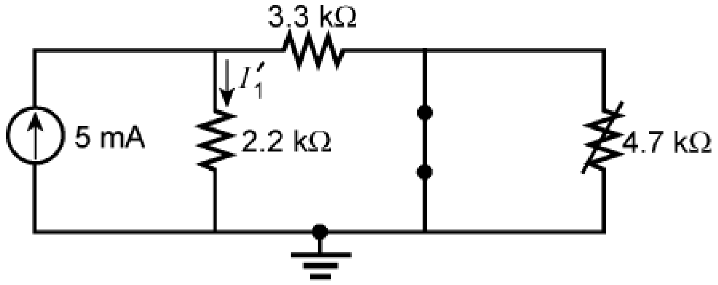


FIG. 9.54
Substituting the Thévenin equivalent circuit for the network external to the resistor R_L of Fig. 9.49.

<u>Ex.</u> Using superposition, find the current through *R*1 for the network in Fig. below:

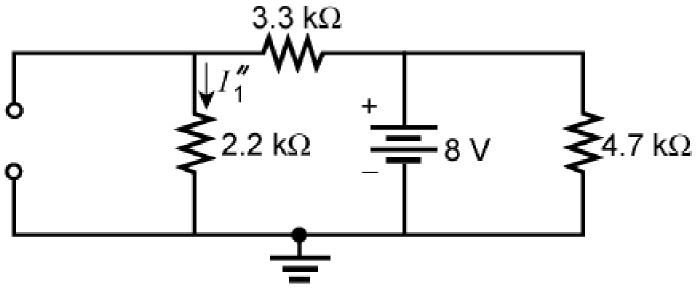


Sol.



$$I_1' = \frac{3.3\,\text{k}\Omega(5\,\text{mA})}{2.2\,\text{k}\Omega + 3.3\,\text{k}\Omega} = \frac{16.5\,\text{mA}}{5.5}$$

$$= 3\,\text{mA}$$



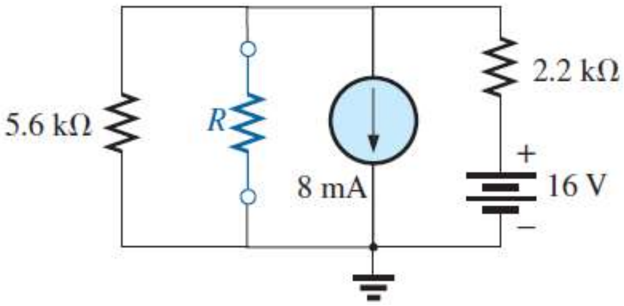
$$I''_1 = \frac{8\,\text{V}}{3.3\,\text{k}\Omega + 2.2\,\text{k}\Omega} = \frac{8\,\text{V}}{5.5\,\text{k}\Omega}$$

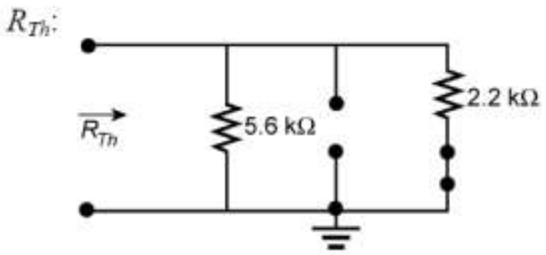
$$= 1.45\,\text{mA}$$

I′ and I′′ in the same direction so:

$$I_1 = I'_1 + I''_1 = 3\,\text{mA} + 1.45\,\text{mA} = \mathbf{4.45\,mA}$$

Ex. Find the Thévenin equivalent circuit for the network external to the resistor R for the network in Fig. below:



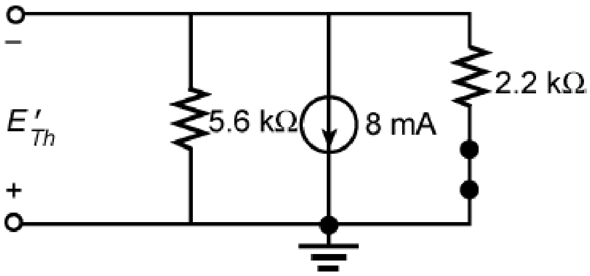


$R_{Th} = 5.6\ \text{k}\Omega \parallel 2.2\ \text{k}\Omega = \mathbf{1.58\ k\Omega}$

$$= \frac{5.6 \times 2.2}{5.6 + 2.2} = 1.58k\Omega$$

E_{Th}: Superposition:

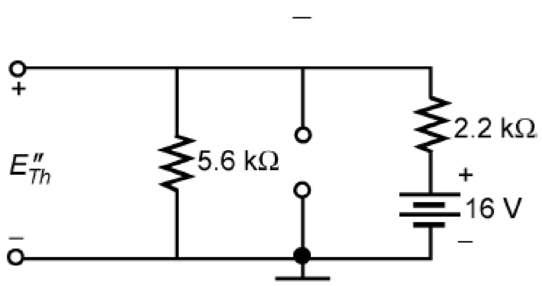
I:



$E'_{Th} = IR_T$

=8 mA×1.58 KΩ= 12.64V

E:

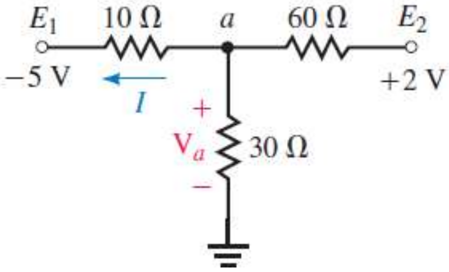


$$E''_{Th} = \frac{5.6\,k\Omega(16\,V)}{5.6\,k\Omega + 2.2\,k\Omega} = 11.49\,V$$

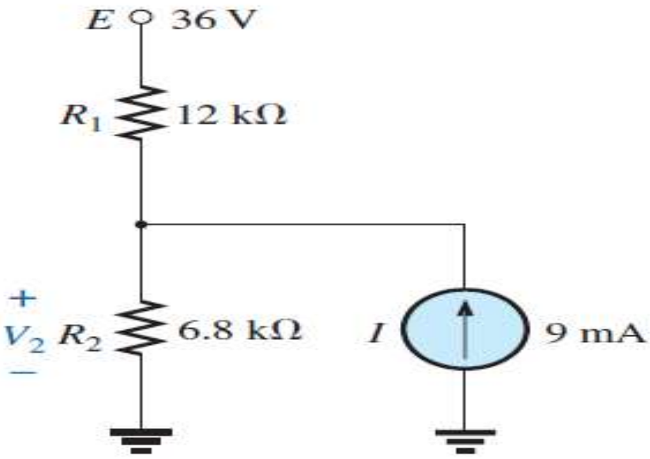
Eth= 12.64 - 11.49

Eth=1.15 V

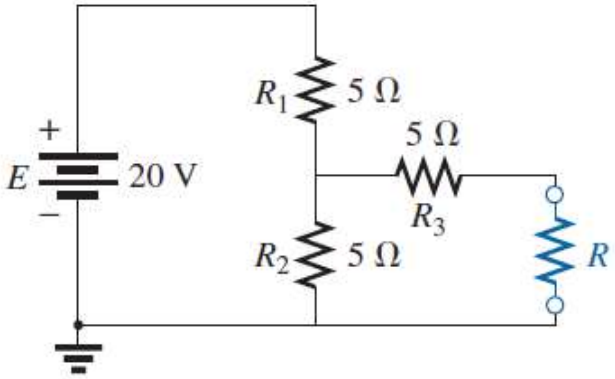
Exercise: Use superposition to solve for the voltage *Va* and the current *I* in the circuit of Figure below:



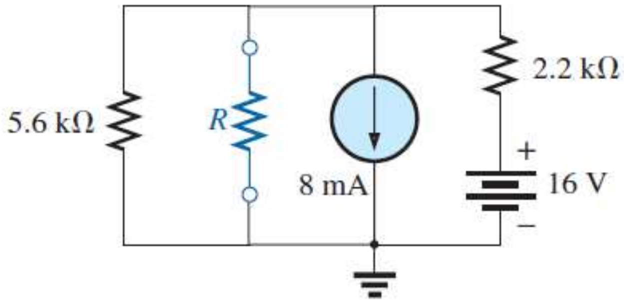
Exercise Using superposition, find the voltage *V2* for the network in fig below:



Exercise: Find the power delivered to *R* when *R* is 2 Ω and 100 Ω.



Exercise Find the Thévenin equivalent circuit for the network external to the resistor *R* for the network in Fig. below



<u>3-NORTON'S THEOREM</u> Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor, as shown in Fig. (a)

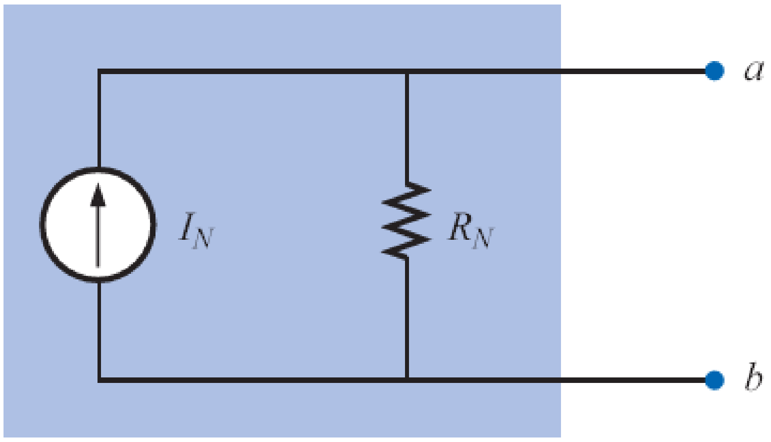


Fig.(a)

1. *Remove that portion of the network across which the Norton equivalent circuit is found.*
2. *Mark the terminals of the remaining two-terminal network.*

R_N:

3. *Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{Th}$, the procedure and value obtained using the approach described for Thévenin's theorem will determine the proper value of R_N.*

I_N:

4. *Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.*

Conclusion:

5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Note The Norton and Thévenin equivalent circuits can also be found from each other by using the source transformation Fig. (b)

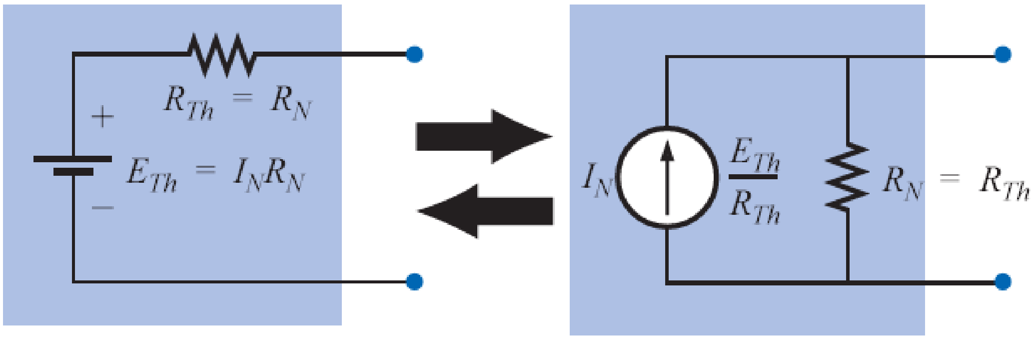
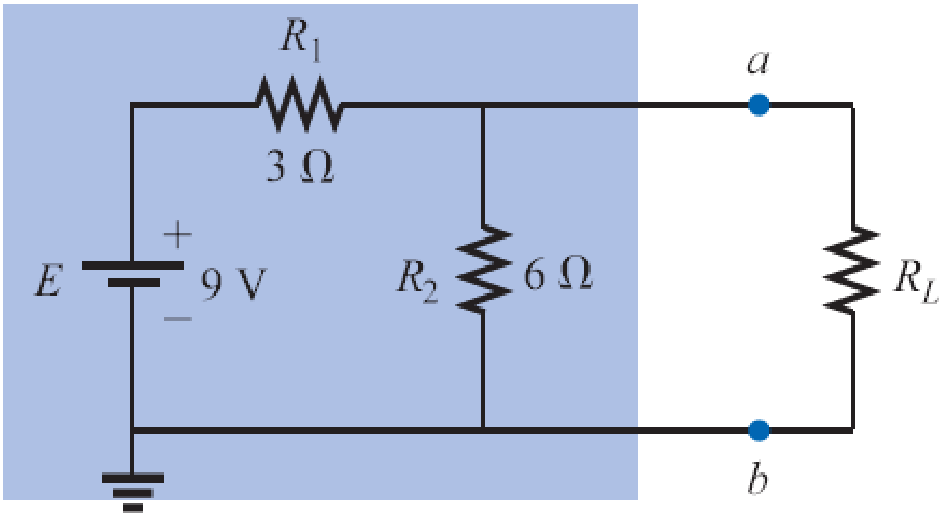


Fig.(b) Converting between Thévenin and Norton equivalent circuits.

EXAMPLE (1) Find the Norton equivalent circuit for the network in the shaded area of Fig. (1)



fig(1)

Solution:

Steps 1 and 2 are shown in Fig. (1-a)

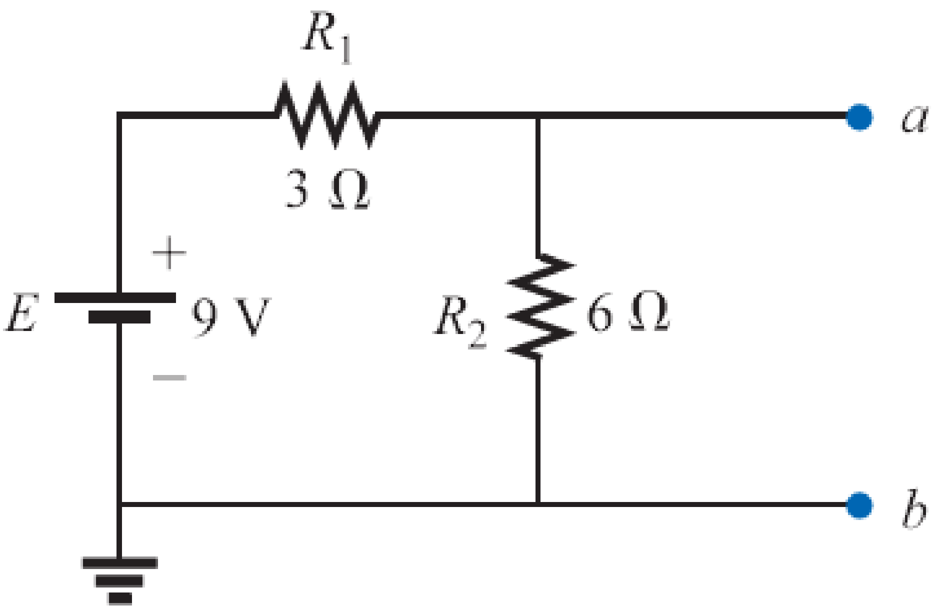


Fig.(1-a)

Step 3 find (R_N) as shown in Fig. (1-b)

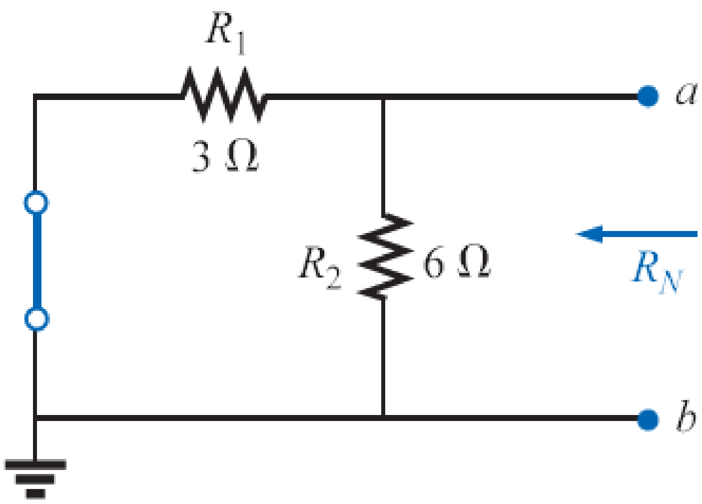
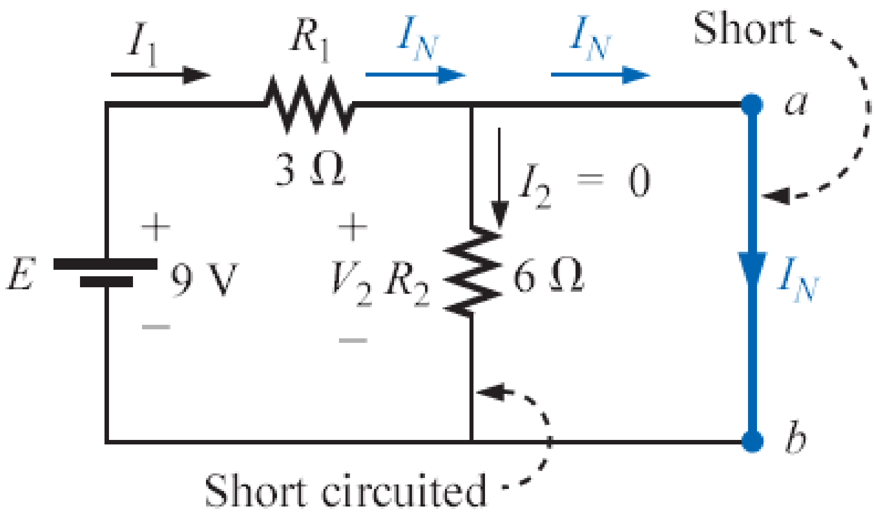


Fig.(1-b) Determining R_N for the network

$$R_N = R_1 \parallel R_2 = 3\ \Omega \parallel 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = \mathbf{2\ \Omega}$$

Step 4 find (I_N) as shown in Fig. (1-c), clearly indicating that the short-circuit connection between terminals (a) and(b) is in parallel with R_2 and eliminates its effect. I_N is therefore the same as through R_1, and the full battery voltage appears across R_1 .



Fig(1-c)

$$V_2 = I_2 R_2 = (0)6\ \Omega = 0\ \text{V}$$

Therefore,

$$I_N = \frac{E}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = \mathbf{3\ A}$$

Step 5: See Fig. (1-d). Substituting the Norton equivalent circuit for the network external to the resistor RL of fig.(1)

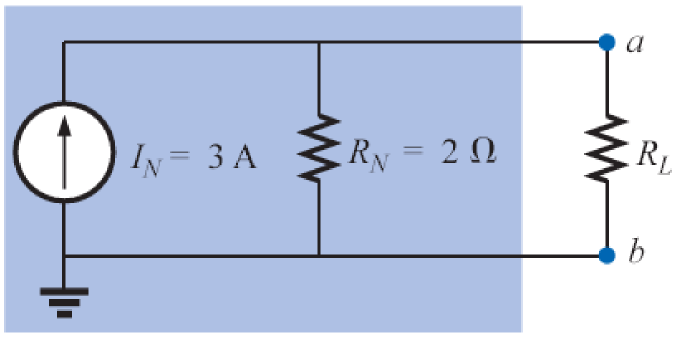


Fig.1-d

Substituting the Norton equivalent circuit for the network external to the resistor RL of fig.(1)

A simple conversion indicates that the Thévenin circuits are, in fact, the same as Norton circuit (Fig. 1-e).

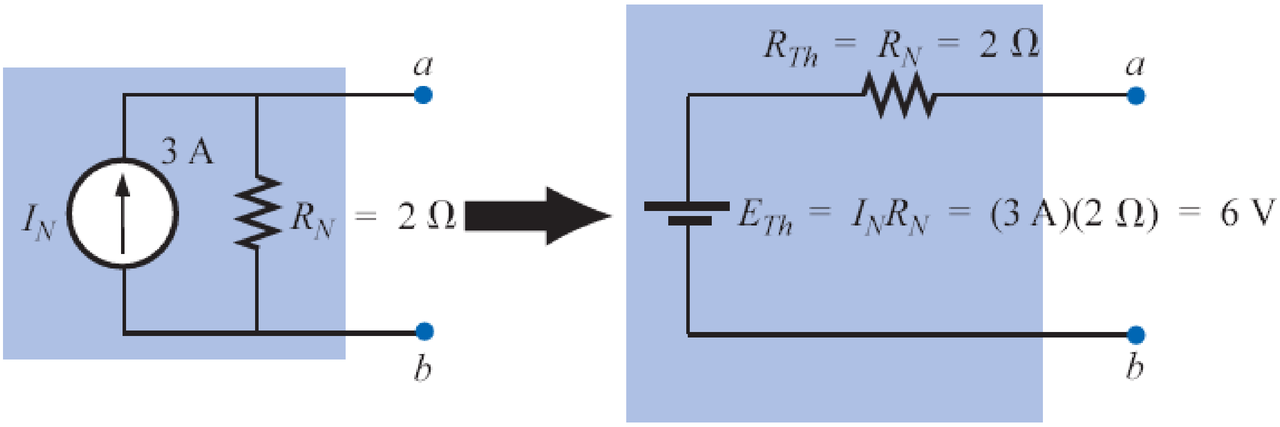


Fig.(1-e)

Converting the Norton equivalent circuit of Fig. (1) To a Thévenin equivalent circuit.

EXAMPLE 2 Find the Norton equivalent external to the 9**Ω** resistor in Fig. (2)

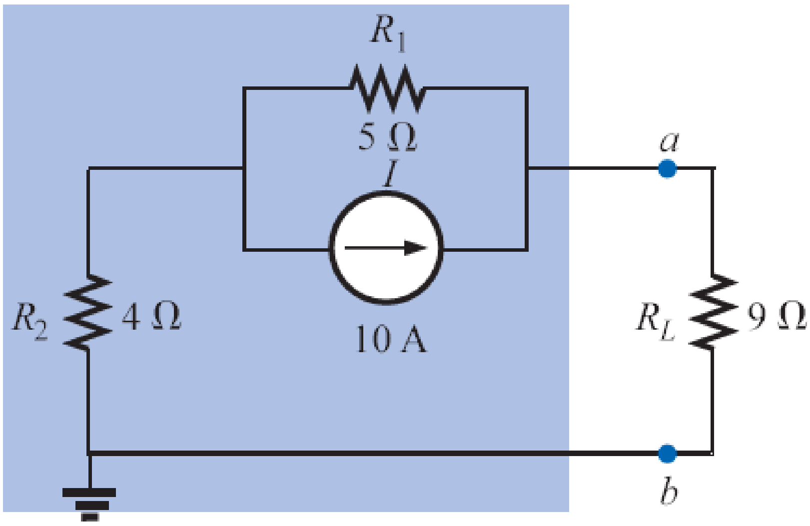
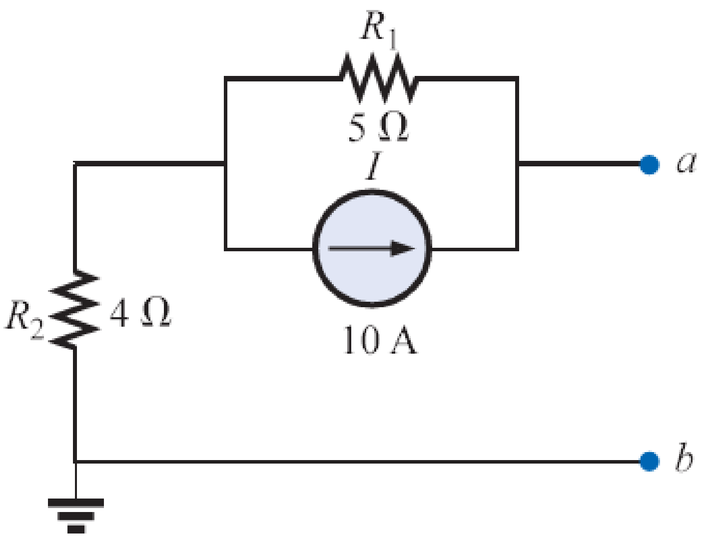


Fig.(2)

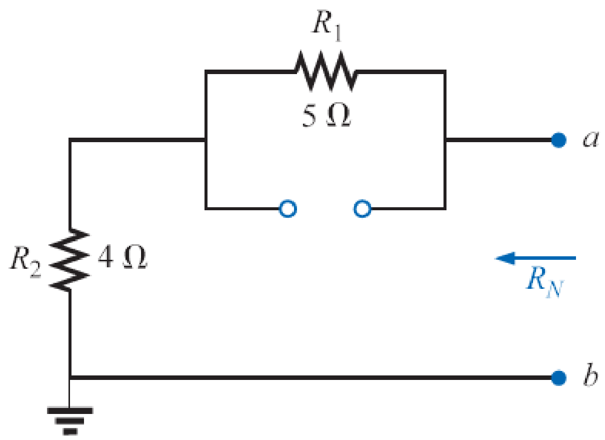
Solution:

Steps 1 and 2: See Fig. (2-a).



Fig(2-a)

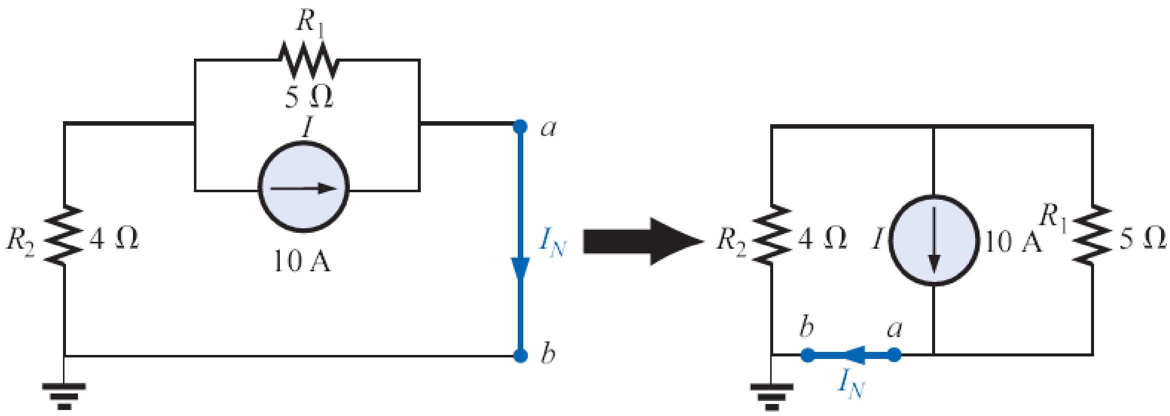
Step 3: find (R_N) See Fig. (2-b),



Fig(2-b)

$$R_N = R_1 + R_2 = 5\ \Omega + 4\ \Omega = \mathbf{9\ \Omega}$$

Step 4: find I_N As shown in Fig. (2-c) the Norton current is the same as the current through the 4**Ω** resistor. Applying the current divider rule,



Fig(2-c) Determining I_N for the network of Fig. (2)

$$I_N = \frac{R_1 I}{R_1 + R_2} = \frac{(5\ \Omega)(10\ \text{A})}{5\ \Omega + 4\ \Omega} = \frac{50\ \text{A}}{9} = \mathbf{5.556\ A}$$

Step 5: See Fig. (2-d)

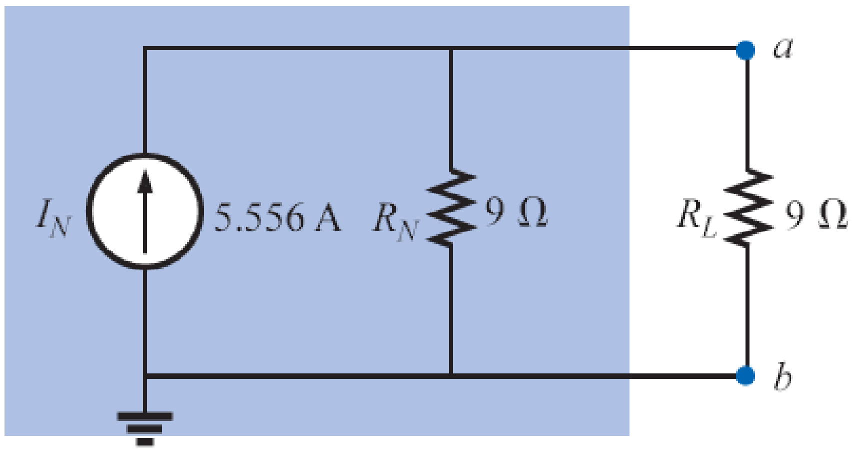


Fig.(2-d)

Substituting the Norton equivalent circuit for the network external to the resistor R_L of Fig. (2).

EXAMPLE (3) (Two sources) Find the Norton equivalent circuit for the portion of the network to the left of a-b in Fig. (3)

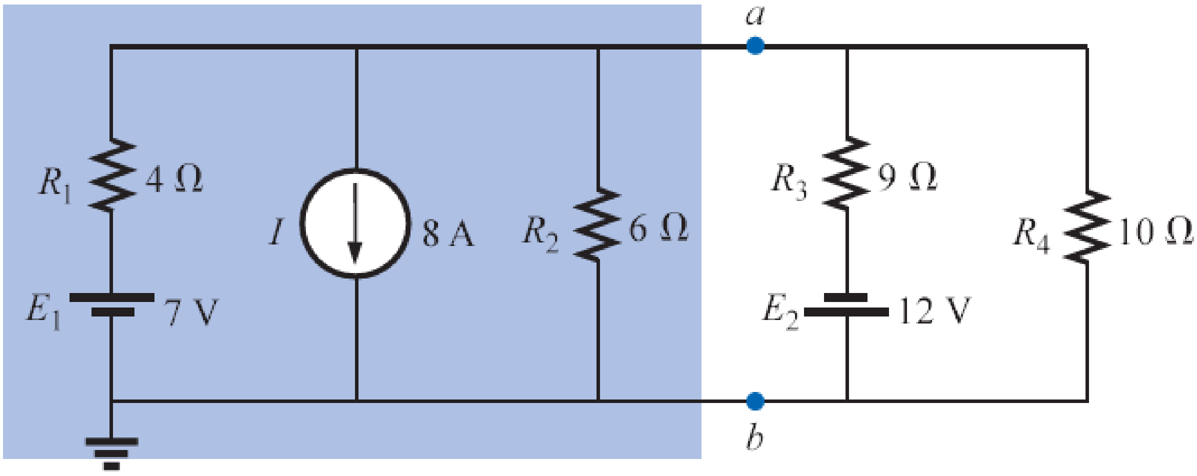


Fig.(3)

Solution:

Steps 1 and 2: See Fig. (3-a)

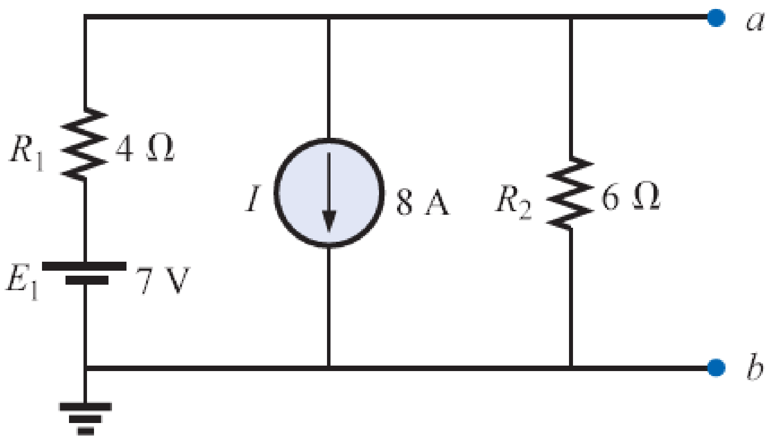


Fig.(3-a)

Step 3 is shown in Fig. (3-b), and

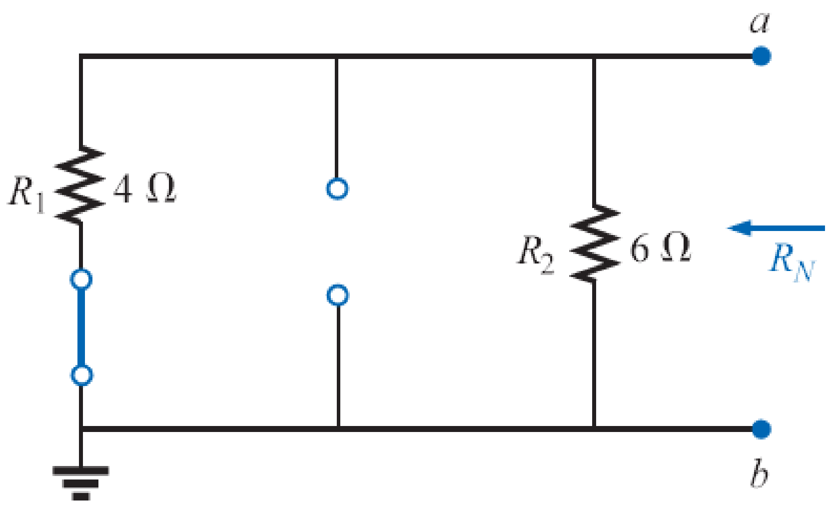


Fig.(3-b)

$$R_N = R_1 \parallel R_2 = 4\ \Omega \parallel 6\ \Omega = \frac{(4\ \Omega)(6\ \Omega)}{4\ \Omega + 6\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$

Step 4: (Using superposition)

For the 7V battery (3-c),

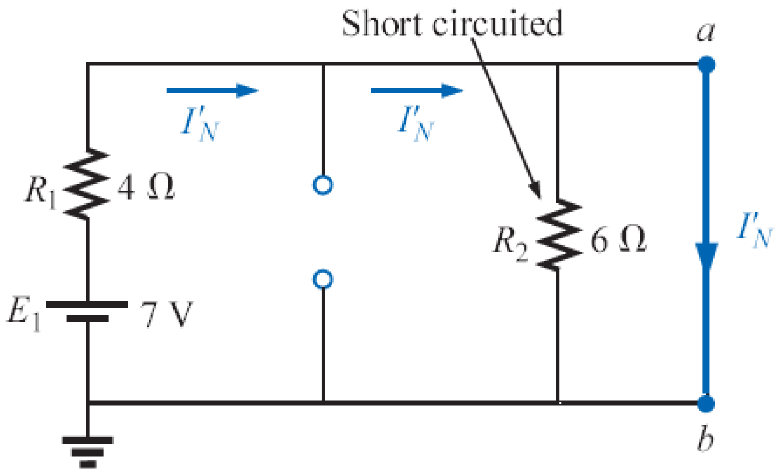


Fig.(3-c) Determining the contribution to I_N from the voltage source E1.

$$I'_N = \frac{E_1}{R_1} = \frac{7\ \text{V}}{4\ \Omega} = 1.75\ \text{A}$$

For the 8 A source (Fig. 3-d) we find that both R1 and R2 have been "short circuited" by the direct connection between a and b, and

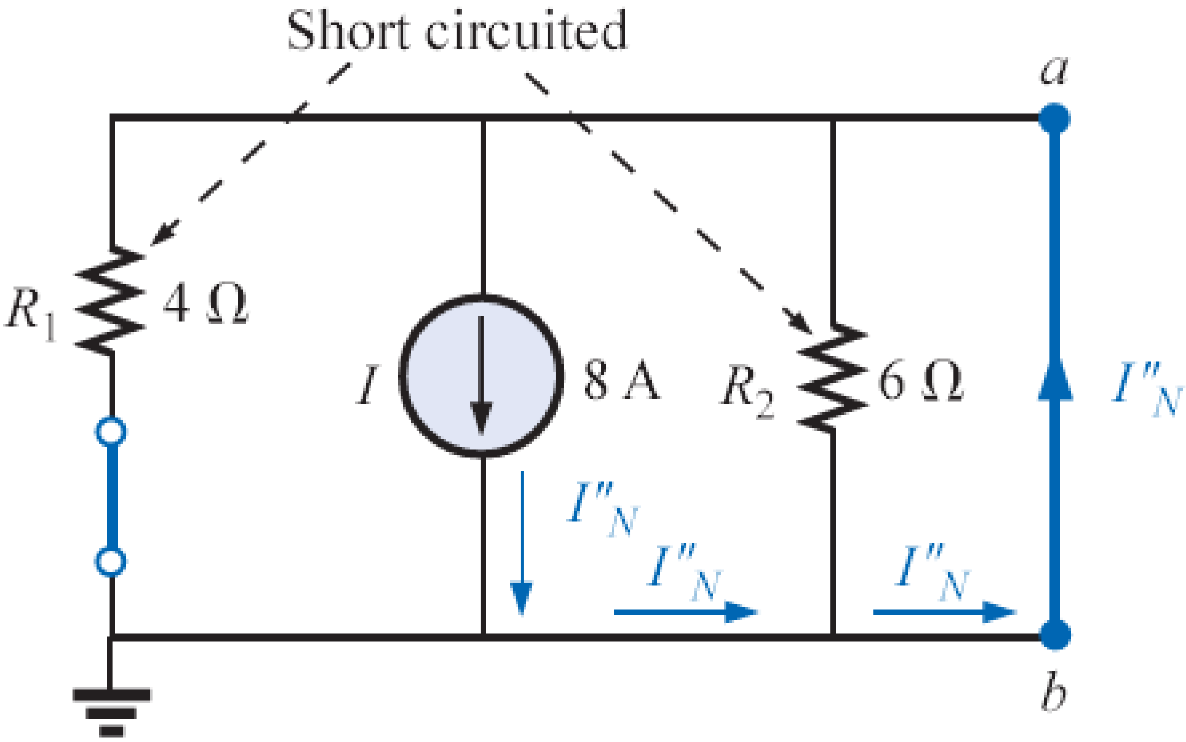


Fig.(3-d) Determining contribution to I_N from the current source I.

$$I''_N = I = 8\text{ A}$$

The result is

$$I_N = I''_N - I'_N = 8\text{ A} - 1.75\text{ A} = \mathbf{6.25\text{ A}}$$

Step 5: See Fig.(3-e).

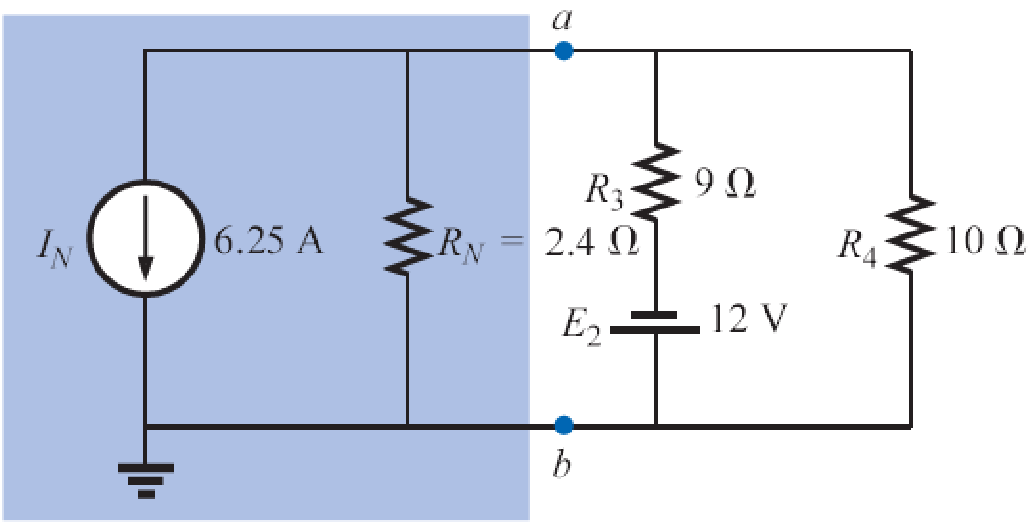


Fig.(3-e)

Substituting the Norton equivalent circuit for the network to the left of terminals (a-b) in Fig. (3).

4-MAXIMUM POWER TRANSFER THEOREM

The maximum power transfer theorem states the following:

A load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thévenin resistance of the network as "seen" by the load .

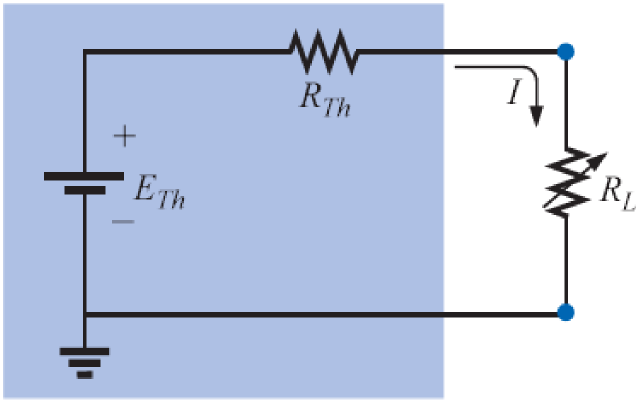


Fig.(a)

Defining the conditions for maximum power to a load using the Thévenin quivalent circuit.

For the network of Fig. (a), maximum power will be delivered to the load when:

$$R_L = R_{Th}$$

For the network of Fig. (a):

$$I = \frac{E_{Th}}{R_{Th} + R_L}$$

and

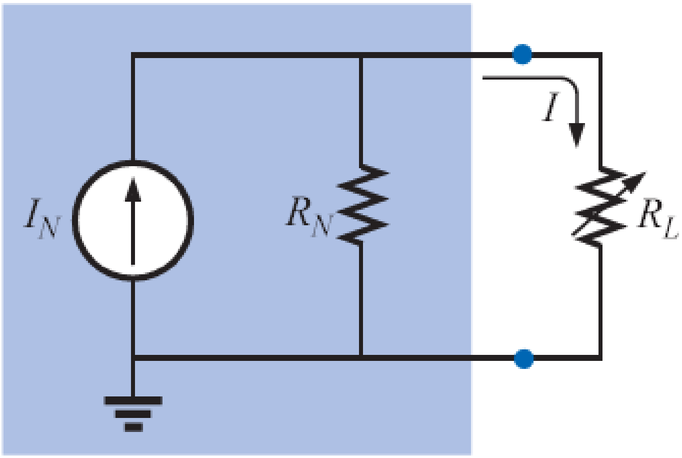
$$P_L = I^2R_L = \left(\frac{E_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

so that

$$P_L = \frac{E_{Th}^2 R_L}{(R_{Th} + R_L)^2}$$

For the Norton equivalent circuit of Fig. (b), maximum power will be delivered to the load when:

$$R_L = R_N$$



Fig(b)

Defining the conditions for maximum power to a load using the Norton equivalent circuit.

The dc operating efficiency of a system is defined by the ratio of the power delivered to the load to the power supplied by the source; that is,

$$\eta\% = \frac{P_L}{P_s} \times 100\%$$

When $R_L = R_{Th}$,

$$\eta\% = \frac{R_L}{R_{Th} + R_L} \times 100\%$$

The power delivered to R_L under maximum power conditions ($R_L = R_{Th}$) is

$$I = \frac{E_{Th}}{R_{Th} + R_L} = \frac{E_{Th}}{2R_{Th}}$$

$$P_L = I^2 R_L = \left(\frac{E_{Th}}{2R_{Th}}\right)^2 R_{Th} = \frac{E_{Th}^2 R_{Th}}{4R_{Th}^2}$$

and

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} \quad \text{(watts, W)}$$

For the Norton circuit of Fig. (b),

$$P_{L_{max}} = \frac{I_N^2 R_N}{4} \quad \text{(W)}$$

Example 4 : A dc generator, battery, and laboratory supply are connected to a resistive load RL in Fig. 4(a), (b), and (c), respectively.

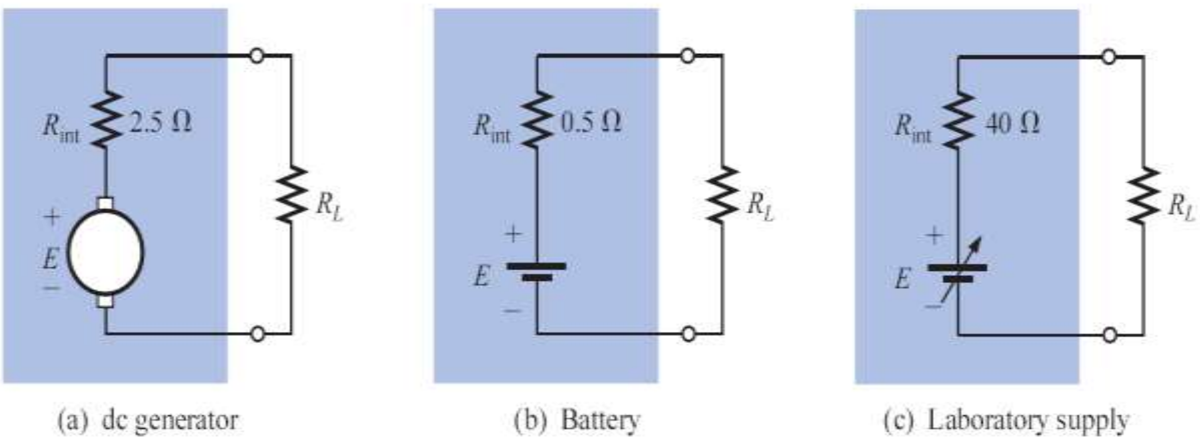


Fig.(4)

a. For each, determine the value of R_L for maximum power transfer to R_L.
b. Determine R_L for 75% efficiency.

Solutions:

a. For the dc generator,

$$R_L = R_{Th} = R_{int} = \mathbf{2.5\ \Omega}$$

For the battery,

$$R_L = R_{Th} = R_{int} = \mathbf{0.5\ \Omega}$$

For the laboratory supply,

$$R_L = R_{Th} = R_{int} = \mathbf{40\ \Omega}$$

b. For the dc generator,

$$\eta = \frac{P_o}{P_s} \qquad (\eta \text{ in decimal form})$$

$$\eta = \frac{R_L}{R_{Th} + R_L}$$

$$\eta(R_{Th} + R_L) = R_L$$

$$\eta R_{Th} + \eta R_L = R_L$$

$$R_L(1 - \eta) = \eta R_{Th}$$

and

$$R_L = \frac{\eta R_{Th}}{1 - \eta}$$

$$R_L = \frac{0.75(2.5\ \Omega)}{1 - 0.75} = \mathbf{7.5\ \Omega}$$

For the battery,

$$R_L = \frac{0.75(0.5\ \Omega)}{1 - 0.75} = \mathbf{1.5\ \Omega}$$

For the laboratory supply,

$$R_L = \frac{0.75(40\ \Omega)}{1 - 0.75} = \mathbf{120\ \Omega}$$

Note:

For loads connected directly to a dc voltage supply, maximum power will be delivered to the load when the load resistance is equal to the internal resistance of the source; that is, when

$$R_L = R_{int}$$ **Or** $$R_L = R_S$$

EXAMPLE 5 : Analysis of a transistor network resulted in the reduced configuration of Fig(5). Determine the R_L necessary to transfer maximum power to RL, and calculate the power of RL under these conditions.

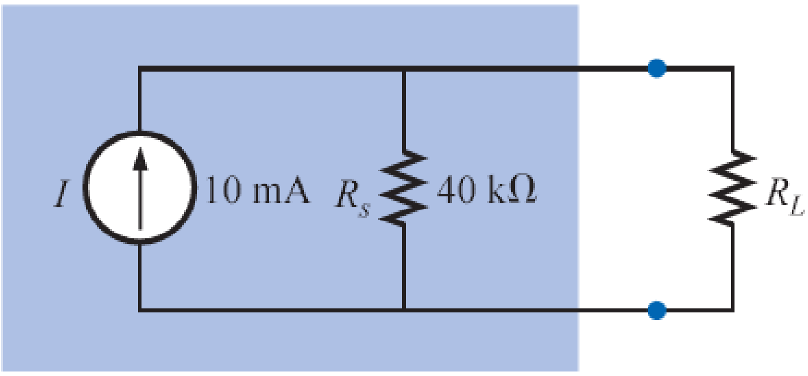


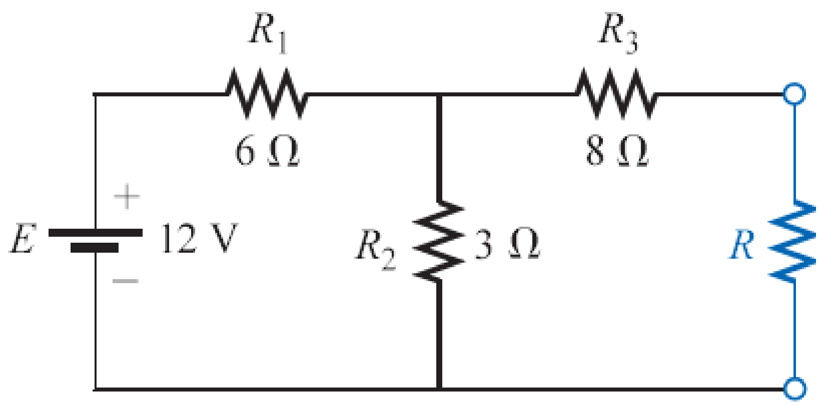
Fig.(5)

Solution:

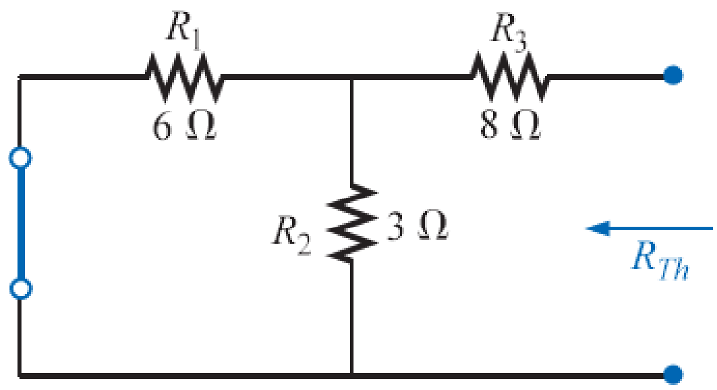
$$R_L = R_s = \mathbf{40\ k\Omega}$$

$$P_{L_{max}} = \frac{I_N^2 R_N}{4} = \frac{(10\ \text{mA})^2(40\ \text{k}\Omega)}{4} = \mathbf{1\ W}$$

EXAMPLE 6 For the network of Fig. (6), determine the value of R for maximum power to R, and calculate the power delivered under these conditions.



Fig(6)



Fig(6-a)

Determining R_{Th} for the network external to the resistor R of Fig(6).

Solution:

$$R_{Th} = R_3 + R_1 \parallel R_2 = 8\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 8\ \Omega + 2\ \Omega$$

$$R = R_{Th} = \mathbf{10\ \Omega}$$

See Fig. (6-b) ,using voltage divider rule to find V_{R2}:

Where $E_{Th} = V_{R2}$

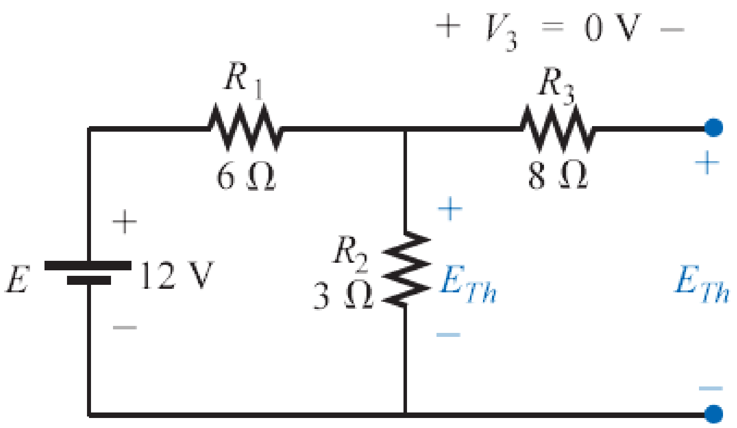


Fig.(6-b)

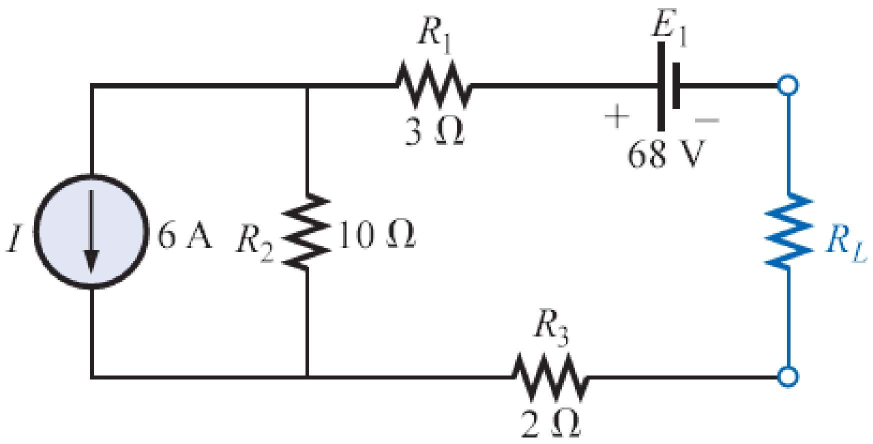
Determining E_{Th} for the network external to the resistor R of Fig. (6)

$$E_{Th} = \frac{R_2 E}{R_2 + R_1} = \frac{(3\ \Omega)(12\ \text{V})}{3\ \Omega + 6\ \Omega} = \frac{36\ \text{V}}{9} = \mathbf{4\ V}$$

Then:

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(4\ \text{V})^2}{4(10\ \Omega)} = \mathbf{0.4\ W}$$

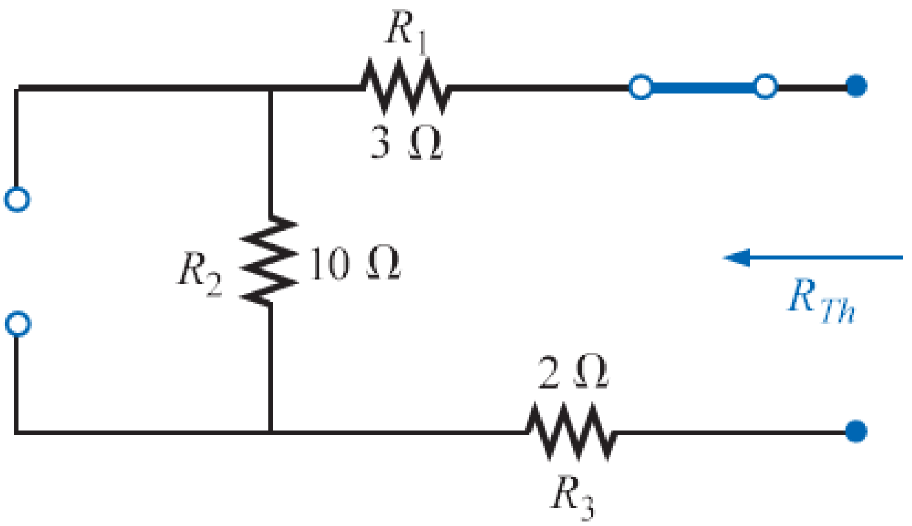
<u>EXAMPLE 7</u> Find the value of R_L in Fig(7) for maximum power to R_L, and determine the maximum power.



Fig(7)

<u>Solution:</u>

See Fig. (7-a).

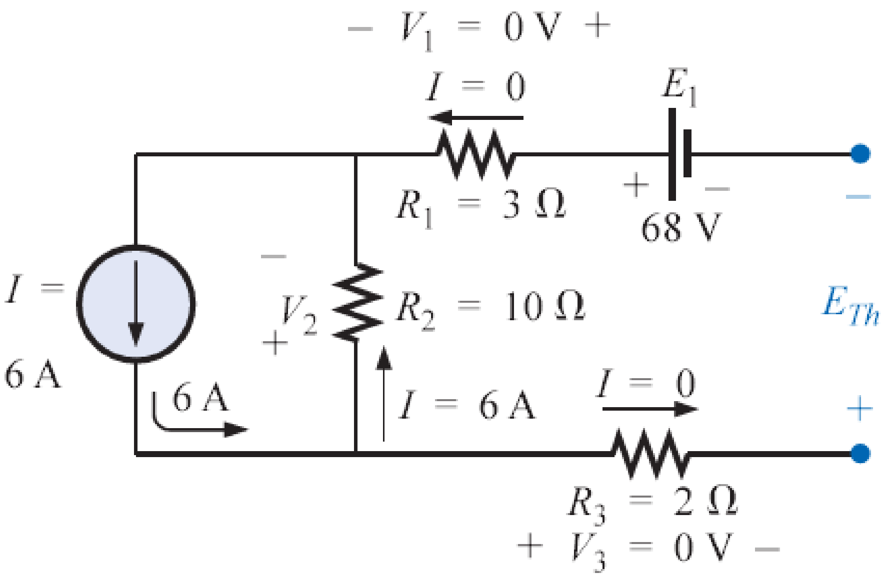


Fig(7-a)

$$R_{Th} = R_1 + R_2 + R_3 = 3\ \Omega + 10\ \Omega + 2\ \Omega = 15\ \Omega$$

and $$R_L = R_{Th} = \mathbf{15\ \Omega}$$

Note Fig. (7-b), where



Fig(7-b) Determining E_{Th} for the network external to the resistor R_L of Fig. (7)

$$V_1 = V_3 = 0\text{ V}$$

and $$V_2 = I_2R_2 = IR_2 = (6\text{ A})(10\ \Omega) = 60\text{ V}$$

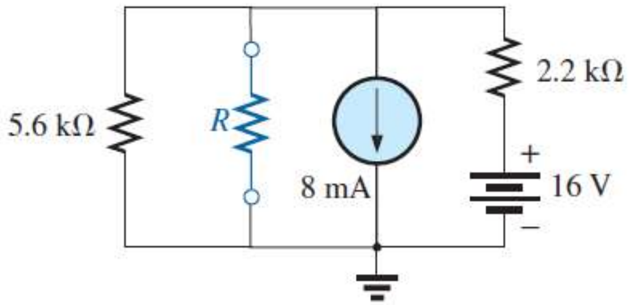
Applying Kirchhoff's voltage law,

$$\Sigma_{\circlearrowright} V = -V_2 - E_1 + E_{Th} = 0$$

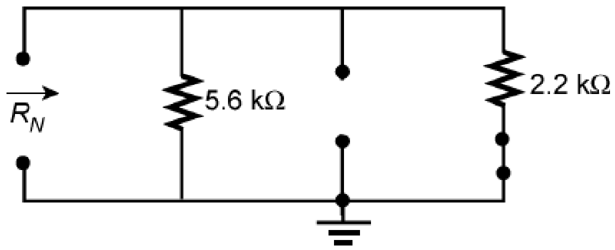
and $$E_{Th} = V_2 + E_1 = 60\text{ V} + 68\text{ V} = 128\text{ V}$$

Thus, $$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128\text{ V})^2}{4(15\ \Omega)} = \mathbf{273.07\ W}$$

Ex. Find the Norton equivalent circuit for the network external to the resistor R for each network in Fig. below:



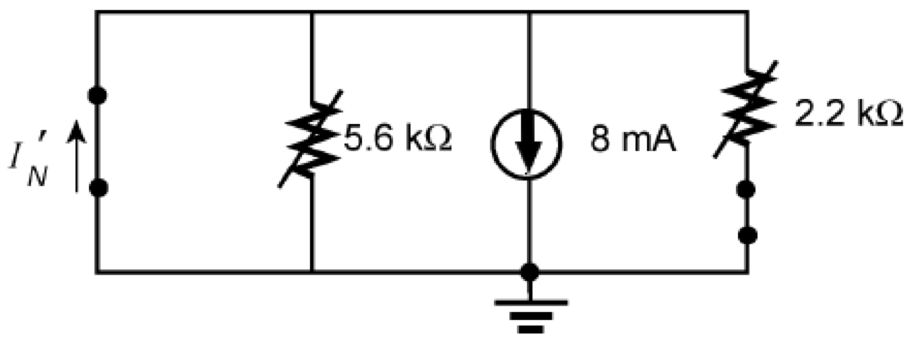
R_N:



$R_N = 5.6\ \text{k}\Omega \parallel 2.2\ \text{k}\Omega = \mathbf{1.58\ k\Omega}$

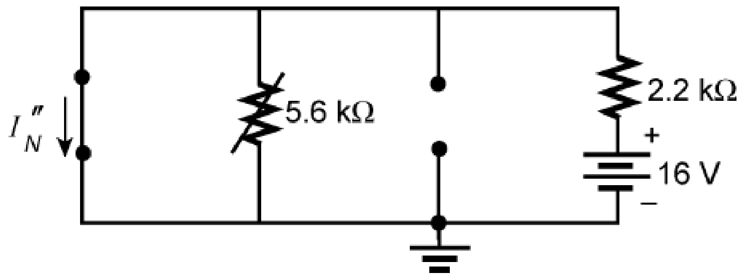
I_N:

I:



$I'_N = 8\ \text{mA}$

E:

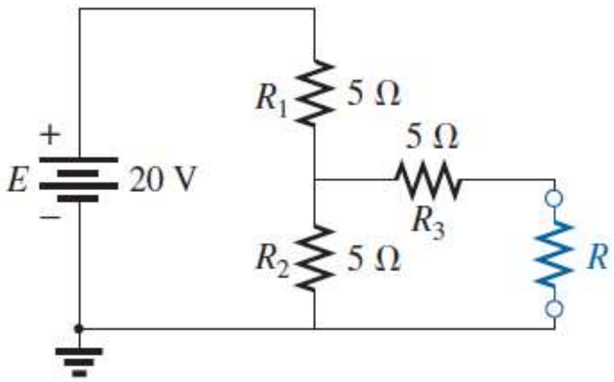


$$I''_N = \frac{16\text{ V}}{2.2\text{ k}\Omega} = 7.27\text{ mA}$$

$$I_N\uparrow = 8\text{ mA} - 7.27\text{ mA} = \mathbf{0.73\text{ mA}}$$

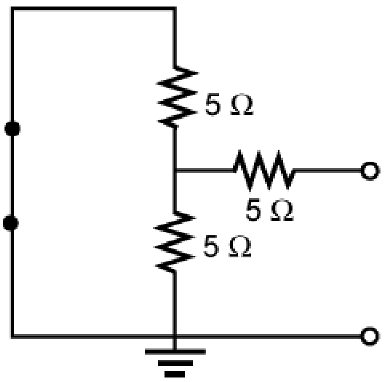
<u>Ex.</u>

a- For network in Fig. below, find the value of *R* for maximum power to *R*.
b- Determine the maximum power to *R* .



<u>Sol.</u>

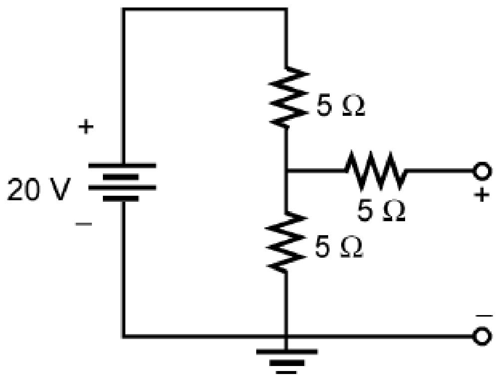
R_{Th}:



$\leftarrow R_{Th} = 5\ \Omega + 5\ \Omega \parallel 5\ \Omega = \mathbf{7.5\ \Omega}$

So : $R=R_{th}=7.5$

E_{Th}:

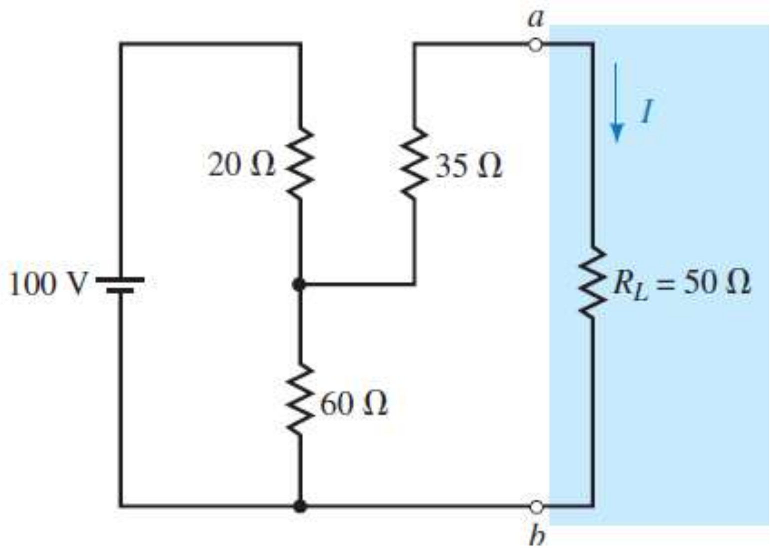


$$E_{Th} = \frac{20\text{ V}}{2} = \mathbf{10\ V}$$

$$P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}}$$

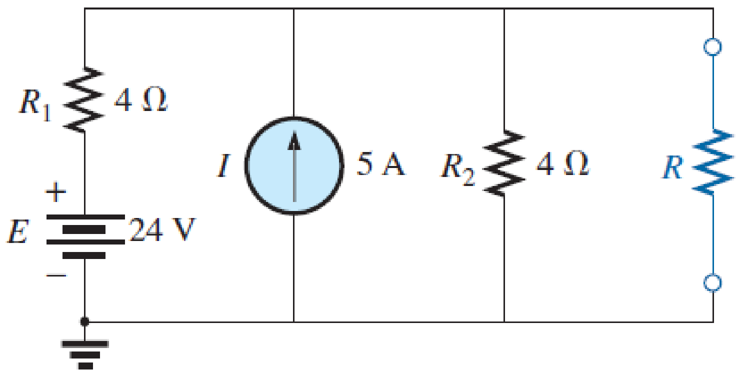
$$P_{L(max)} = \frac{E_{th}^2}{4\times R_{th}} = \frac{10^2}{4\times 7.5} = 3.33\text{W}$$

Exercise : Find the Norton equivalent circuit external to the indicated terminals of Figure below:



Exercise: a- For the network in Fig. below, determine the value of *R* for maximum power to *R*.

b**.** Determine the maximum power to *R*.



MAGNETIC FIELDS In the region surrounding a permanent magnet there exists a magnetic field, which can be represented by **magnetic flux lines** . The strength of a magnetic field in a particular region is directly related to the density of flux lines in that region .If a nonmagnetic material, such as glass or copper, is placed in the flux paths surrounding a permanent magnet, there will be an almost unnoticeable change in the flux distribution (Fig. 1). However, if a magnetic material, such as soft iron, is placed in the flux path, the flux lines will pass through the soft iron rather than the surrounding air because flux lines pass with greater ease through magnetic materials than through air. This principle is put to use in the shielding of sensitive electrical elements and instruments that can be affected by stray magnetic fields (Fig. 2).

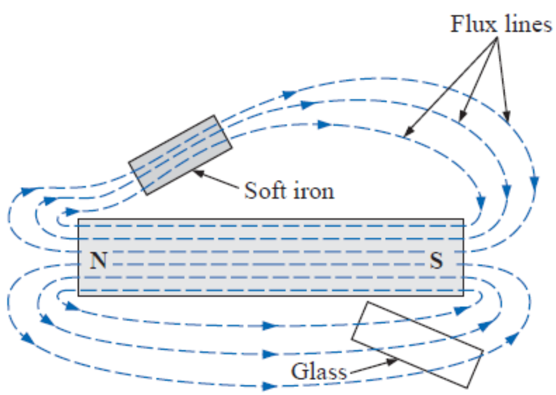


Fig.(1)

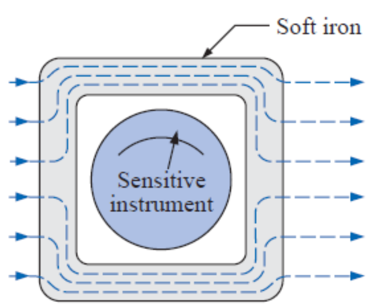


Fig.(2)

The direction of the magnetic flux lines can be found simply by placing the thumb of the *right* hand in the direction of *conventional* current flow and noting the direction of the fingers. (This method is commonly called the *right-hand rule.*) If the conductor is wound in a single-turn coil (Fig. 3), the resulting flux will flow in a common direction through the center of the coil. A coil of more than one turn would produce a magnetic field that would exist in a continuous path through and around the coil (Fig4).

Fig.(3)

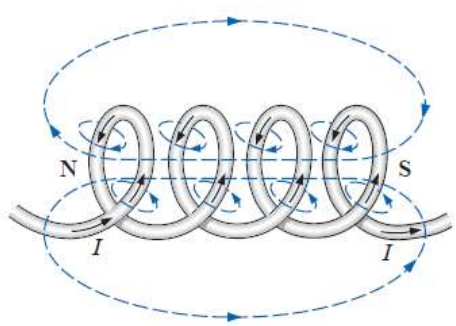


Fig.(4)

The flux distribution of the coil is quite similar to that of the permanent magnet. The flux lines leaving the coil from the left and entering to the right simulate a north and a south pole, respectively. The principal difference between the two flux distributions is that the flux lines are more concentrated for the permanent magnet than for the coil. Also, since the strength of a magnetic field is determined by the density of the flux lines, the coil has a weaker field strength. The field strength of the coil can be effectively increased by placing certain materials, such as iron, steel, or cobalt, within the coil to increase the flux density within the coil. By increasing the field strength with the addition of the core, we have devised an *electromagnet* (Fig. 5)

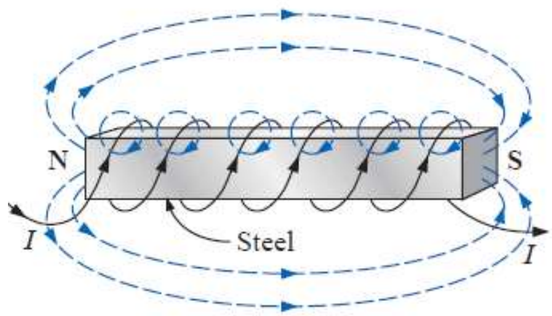


Fig.(5)

The direction of flux lines can be determined for the electromagnet (or in any core with a wrapping of turns) by placing the fingers of the right hand in the direction of current flow around the core. The thumb will then point in the direction of the north pole of the induced magnetic flux, as demonstrated in Fig. 6(a). A cross section of the same electromagnet is included as Fig. 6(b) to introduce the convention for directions perpendicular to the page. The cross and dot refer to the tail and head of the arrow

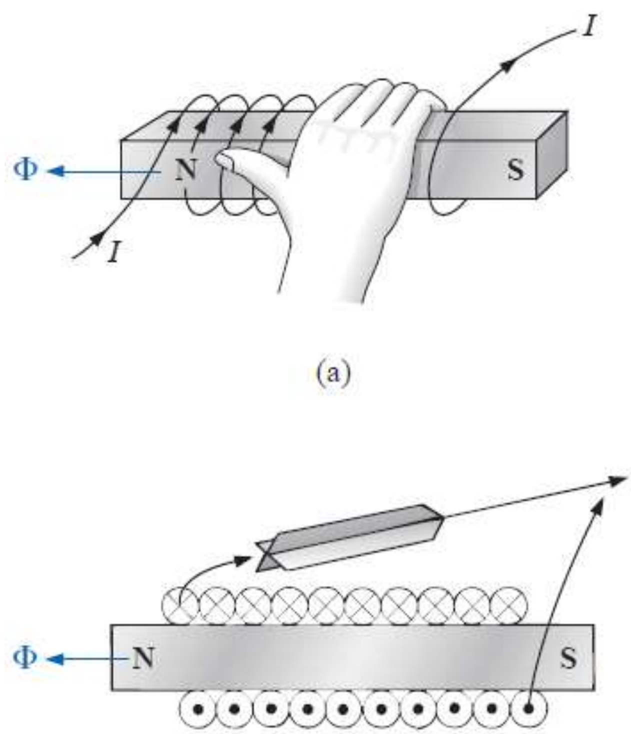


Fig.(6)

Flux density

In the SI system of units, magnetic flux is measured in ***webers*** and has the symbol Φ. The number of flux lines per unit area is called the **flux density *B*.**

$$B = \frac{\Phi}{A}$$

B = teslas (T)
Φ = webers (Wb)
A = square meters (m^2)

PERMEABILITY

The **permeability** (μ*)* of a material, therefore, is a measure of the ease with which magnetic flux lines can be established in the material. It is similar in many respects to conductivity in electric circuits. The permeability of free
space μ*o* (vacuum) is :

$$\mu_o = 4\pi \times 10^{-7} \frac{\text{Wb}}{\text{A}\cdot\text{m}}$$

the permeability of all **nonmagnetic materials**, such as copper, aluminum, wood, glass, and air, is **the same as that for free space**. Materials that have **permeabilities slightly less than that of free space** are said to be **diamagnetic,** and those with permeabilities slightly greater than that of free space are said to be **paramagnetic.** Magnetic materials, such as iron, nickel, steel, cobalt, and alloys of these metals, have permeabilities hundreds and even thousands of times that of free space. Materials with these very high permeabilities are referred to as **ferromagnetic.**
The ratio of the permeability of a material to that of free space is called its **relative permeability;** that is,

$$\mu_r = \frac{\mu}{\mu o}$$

RELUCTANCE(الممانعة المغناطيسية)

The reluctance, is inversely proportional to the permeability, therefore, materials with high permeability, such as the ferromagnetic, have very small reluctances and will result in an increased measure of flux through the core

$$\mathcal{R} = \frac{l}{\mu A} \qquad \text{(rels, or At/Wb)}$$

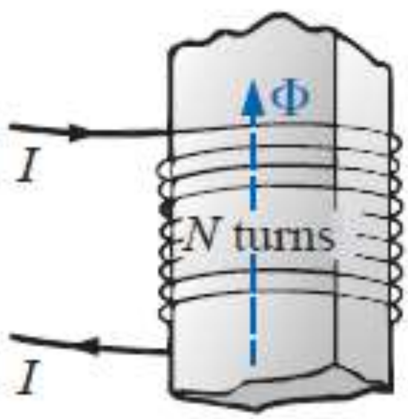
OHM'S LAW FOR MAGNETIC CIRCUITS

For magnetic circuits, the effect desired is the flux Φ The cause is the **magnetomotive force (mmf)** F, which is the external force (or "pressure") required to set up the magnetic flux lines within the magnetic material. The opposition to the setting up of the flux Φ is the reluctance R. .
Substituting, we have :

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}}$$

The **MAGNETO MOTIVE FORCE** $\mathcal{F}$ is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire (Fig.7). In equation form,

$$\mathcal{F} = NI \qquad \text{(ampere-turns, At)}$$



Fig(7)

MAGNETIZING FORCE

The magneto motive force per unit length is called the **magnetizing force** (*H*). In equation form

$$H = \frac{\mathscr{F}}{l} \qquad \text{(At/m)}$$

Substituting for the magnetomotive force will result in

$$H = \frac{NI}{l} \qquad \text{(At/m)}$$

the magnetizing force is independent of the type of core material—it is determined solely by the number of turns, the current and the length of the core. As the magnetizing force increases, the permeability rises to a maximum and then drops to a minimum.
The flux density and the magnetizing force fig.(8)and (8-a)are related by the following equation:

$$B = \mu H$$

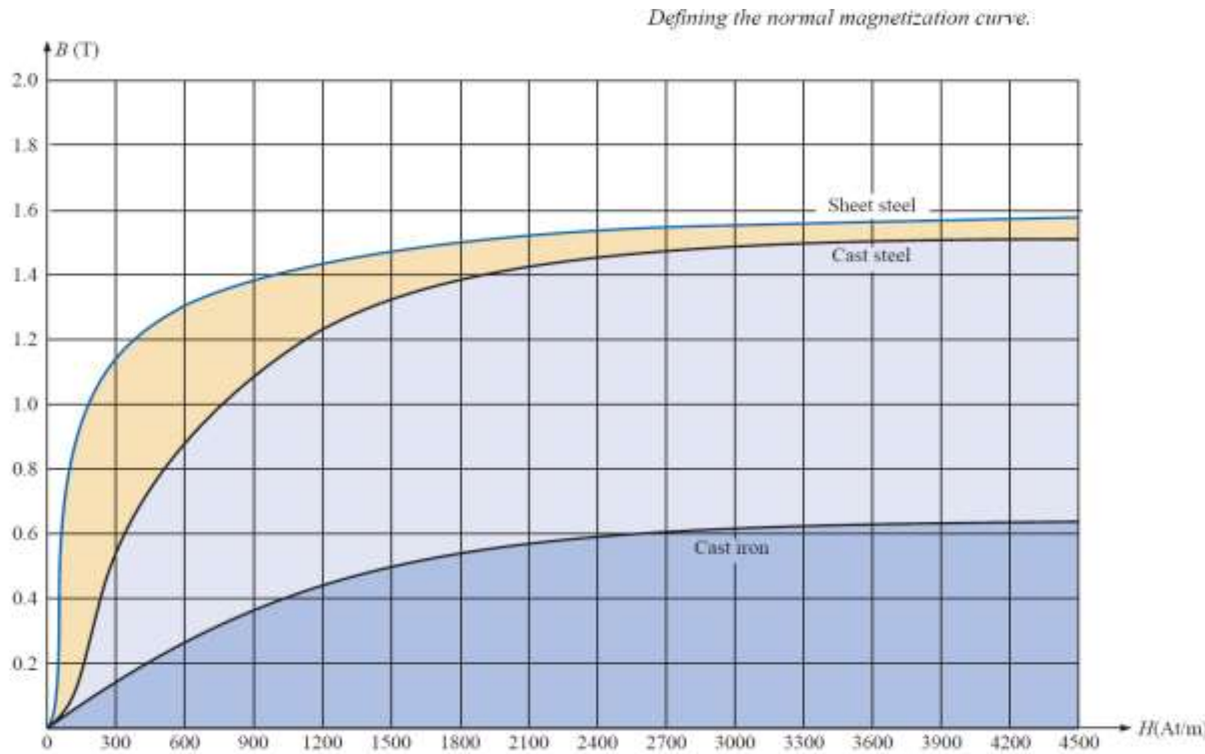


Fig.(8) illustrate the magnetizing force region or (B-H) curve.

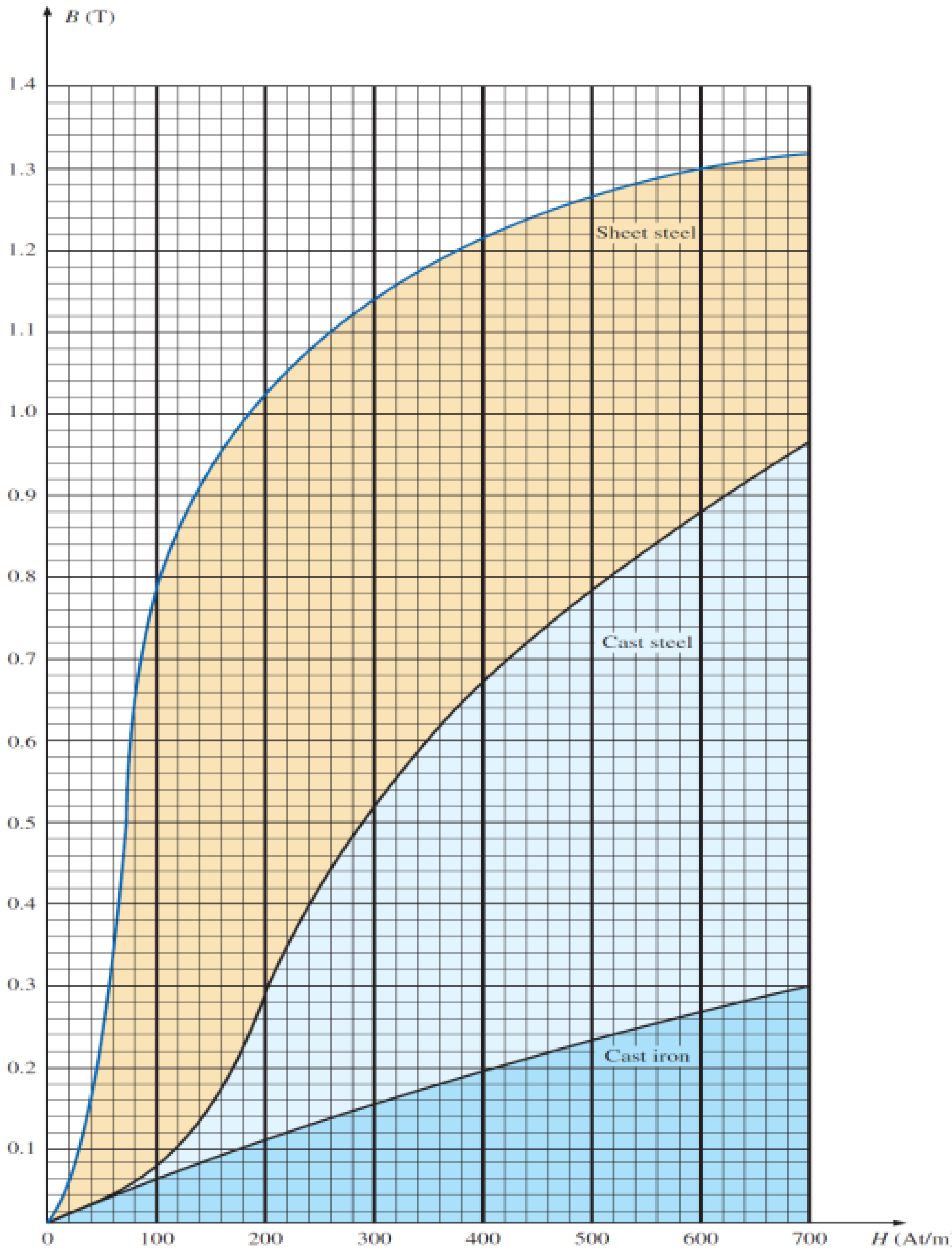


Fig.(8-a) Expanded view of Fig. (8) for the low magnetizing force region

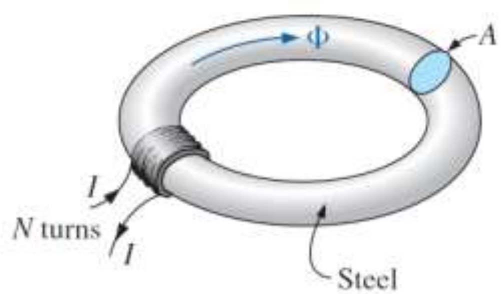
HYSTERESIS

A typical *B-H* curve for a ferromagnetic material such as steel can be derived using the setup in Fig. (9) .The core is initially un-magnetized, and the current $I = 0$. If the current *I* is increased to some value above zero, the magnetizing force *H* increases to a value determined by

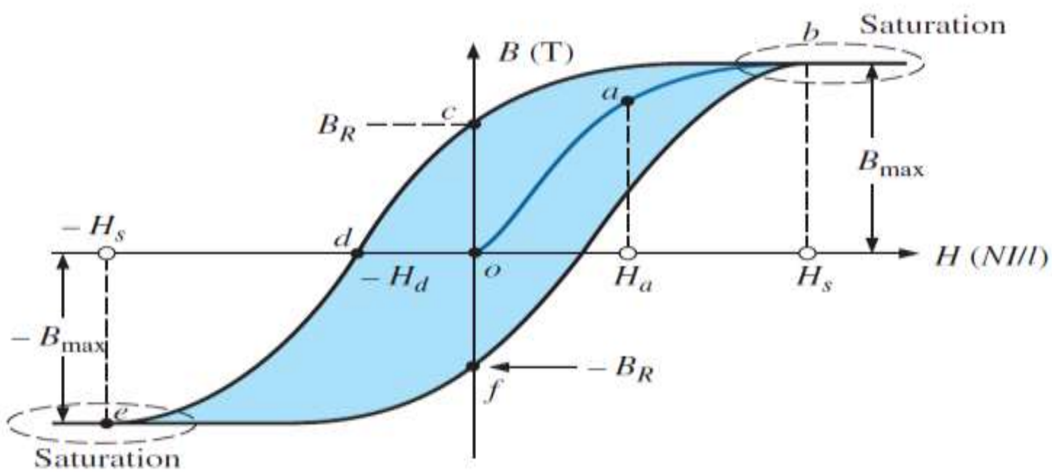
$$H\uparrow = \frac{NI\uparrow}{l}$$

The flux Φ and the flux density *B* ($B=\Phi/A$) also increase with the current *I* (or *H*). If the material has no residual magnetism, and the magnetizing force *H* is increased from zero to some value *Ha* , the *B-H* curve follows the path shown in Fig. (10) between *o* and *a*. If the magnetizing force *H* is increased until saturation (*Hs*) occurs, the curve continues as shown in the figure to point *b*. When saturation occurs, the flux density has, *for all practical purposes*, reached its maximum value. Any further increase in current through the coil increasing $H = NI/l$ results in a very small increase in flux density *B*. If the magnetizing force is reduced to zero by letting *I* decrease to zero, the curve follows the path of the curve between *b* and *c*. The flux density *BR* , which remains when the magnetizing force is zero, is called
the *residual flux density*. It is this residual flux density that makes it possible
to create permanent magnets. If the coil is now removed from the core in Fig. (9), the core will still have the magnetic properties determined by the residual flux density, a measure of its "retentivity." If the current *I* is reversed, developing a magnetizing force, –*H*, the flux density *B* decreases with an increase in *I*. Eventually, the flux density will be zero when -*Hd* (the portion of curve from *c* to *d*) is reached. The magnetizing force -*Hd* required to "coerce" the flux density to reduce its level to zero is called the *coercive force*, a measure of the coercivity of

the magnetic sample. As the force -*H* is increased until saturation again occurs and is then reversed and brought back to zero, the path *def* results. If the magnetizing force is increased in the positive direction (-*H*), the curve traces the path shown from *f* to *b*. The entire curve represented by (*bcdefb)* is called the **hysteresis** curve for the ferromagnetic material fig.(10).



Fig(9)



Fig(10)

AMPÈRE'S CIRCUITAL LAW

States that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop. Sources of mmf are expressed by the equation:

$$\mathscr{F} = NI$$

The equation for the mmf drop across a portion of a magnetic circuit:

$$\mathscr{F} = \Phi\mathscr{R}$$

A more practical equation for the mmf drop is :

$$\mathscr{F} = Hl$$

$$\sum_{\circlearrowright} \mathscr{F} = 0$$

This can be rewrittten as

$$\sum_{\circlearrowright} NI = \sum_{\circlearrowright} H\ell \quad \text{At}$$

So:

NI=Hℓ

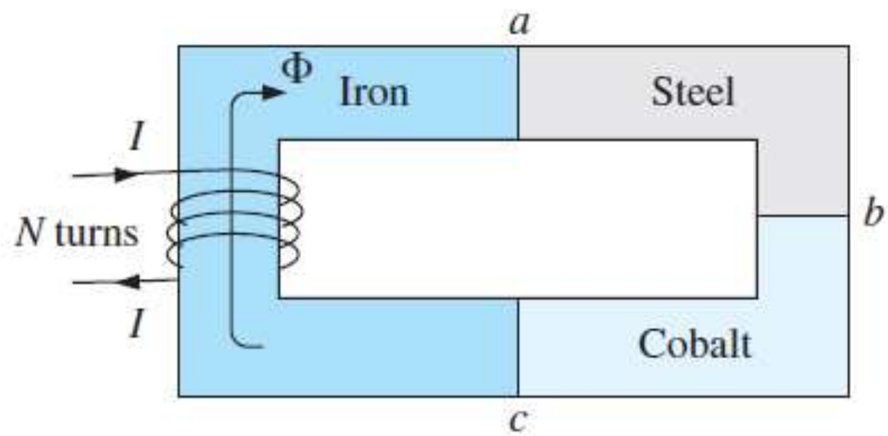
I= current

ℓ= length of magnetic circuit

H=magnetizing force

N= number of turns

Example (Series magnetic circuit of three different materials).



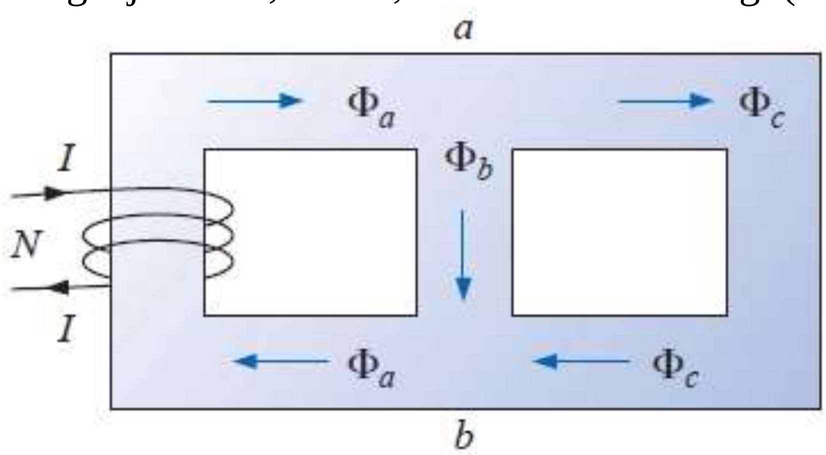
$$\Sigma_{\circlearrowright}\ \mathscr{F} = 0$$

$$\underbrace{+NI}_{\text{Rise}} - \underbrace{H_{ab}l_{ab}}_{\text{Drop}} - \underbrace{H_{bc}l_{bc}}_{\text{Drop}} - \underbrace{H_{ca}l_{ca}}_{\text{Drop}} = 0$$

$$\underbrace{NI}_{\substack{\text{Impressed}\\ \text{mmf}}} = \underbrace{H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}}_{\text{mmf drops}}$$

THE FLUX Φ

the sum of the fluxes entering a junction is equal to the sum of the fluxes leaving a junction; that is, for the circuit of Fig. (11)



Fig(11)

$$\Phi_a = \Phi_b + \Phi_c \qquad \text{(at junction } a\text{)}$$

or

$$\Phi_b + \Phi_c = \Phi_a \qquad \text{(at junction } b\text{)}$$

both of which are equivalent.

SERIES MAGNETIC CIRCUITS DETERMINING (*N*I)*

to solve a magnetic circuit problems, which are basically of two types:

1- mmf (*NI)* must be computed.
This is the type of problem encountered in the design of motors, generators, and transformers

2- *NI* is given, and the flux ϕ of the magnetic circuit must be found. This type of problem is encountered primarily in the design of magnetic amplifiers

For magnetic circuits, the level of *B* or *H* is determined from the other using the *B-H* curve, and μ is seldom calculated unless asked for.
نادراً ما

An approach frequently employed in the analysis of magnetic circuits is the ***table* method.**

Before a problem is analyzed in detail, a table is prepared listing in the extreme left-hand column the various sections of the magnetic circuit. The columns on the right are reserved for the quantities to be found for each section.
We will consider only *series* magnetic circuits in which the flux ϴ is the same throughout.
EXAMPLE (1) For the series magnetic circuit of Fig(12):
a. Find the value of *I* required to develop a magnetic flux of $\phi = 4 \times 10^{-4}$ Wb.
b. Determine μ and μ*r* for the material under these conditions.

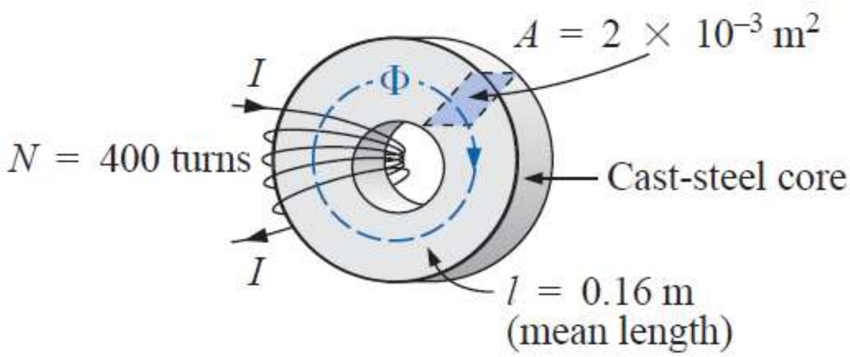


Fig.(12)

Solutions: The magnetic circuit can be represented by the system shown in Fig. 13(a). The electric circuit analogy is shown in Fig. 13(b). Analogies of this type can be very helpful in the solution of magnetic circuits. Table (1) is for part (a) of this problem. The table is fairly trivial for this example, but it does define the quantities to be found.

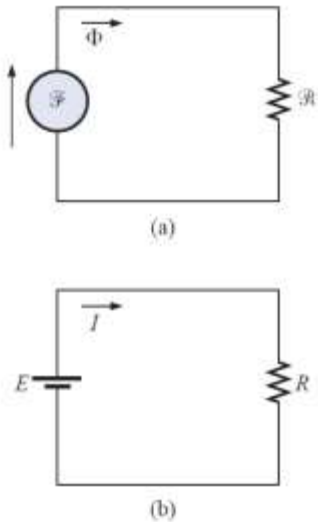


Fig.(13)

Section	Φ (Wb)	A (m²)	B (T)	H (At/m)	l (m)	Hl (At)
One continuous section	4×10^{-4}	2×10^{-3}			0.16	

Table (1)

a. The flux density B is

$$B = \frac{\Phi}{A} = \frac{4 \times 10^{-4}\ \text{Wb}}{2 \times 10^{-3}\ \text{m}^2} = 2 \times 10^{-1}\ \text{T} = 0.2\ \text{T}$$

Using the *B*-*H* curves of Fig. (8-a), we can determine the magnetizing force *H*:

$$H\,(\text{cast steel}) = 170\ \text{At/m}$$

Applying Ampère's circuital law yields

$$NI = Hl$$

and $$I = \frac{Hl}{N} = \frac{(170\ \text{At/m})(0.16\ \text{m})}{400\ \text{t}} = \mathbf{68\ mA}$$

(Recall that t represents turns.)

b-

$$\mu = \frac{B}{H} = \frac{0.2\ \text{T}}{170\ \text{At/m}} = \mathbf{1.176 \times 10^{-3}\ Wb/A\cdot m}$$

and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = \mathbf{935.83}$$

EXAMPLE (2) Calculate the magnetic flux ϕ for the magnetic circuit of Fig. (14).

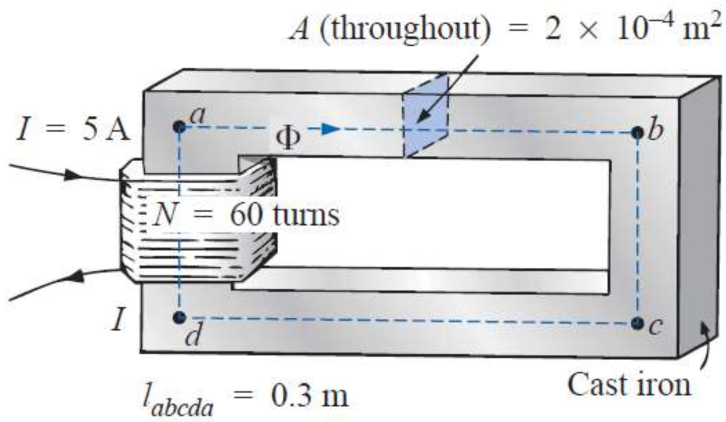


Fig.(14)

Solution: By Ampère's circuital law,

$$NI = H_{abcda} l_{abcda}$$

or

$$H_{abcda} = \frac{NI}{l_{abcda}} = \frac{(60\ \text{t})(5\ \text{A})}{0.3\ \text{m}}$$

$$= \frac{300\ \text{At}}{0.3\ \text{m}} = 1000\ \text{At/m}$$

B (abcda) (from B-H Curve) = 0.39 T

$B = \frac{\phi}{A}$

So $\phi = B \times A$

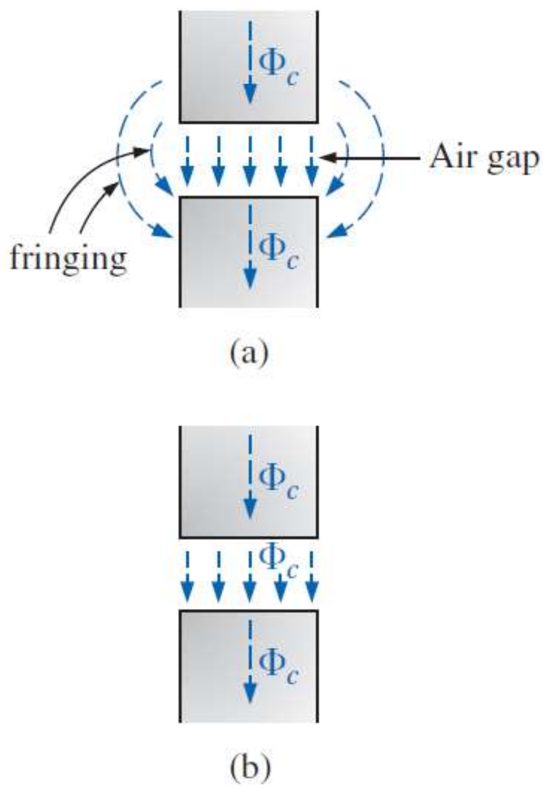
$\phi = 0.39 \times 2 \times 10^{-4}$

$\phi = 0.78$ wb

AIR GAPS

The spreading of the flux lines outside the common area of the core for the air gap in Fig. 15 (a) is known as *fringing.* For our purposes, we shall ignore this effect and assume the flux distribution to be as in Fig. 15. (b) The flux density of the air gap in Fig. 15 (b) is given by

$$B_g = \frac{\Phi_g}{A_g}$$



Fig(15) Air gaps: (a) with fringing; (b) ideal.

where, for our purposes,

$$\Phi_g = \Phi_{core}$$

and

$$A_g = A_{core}$$

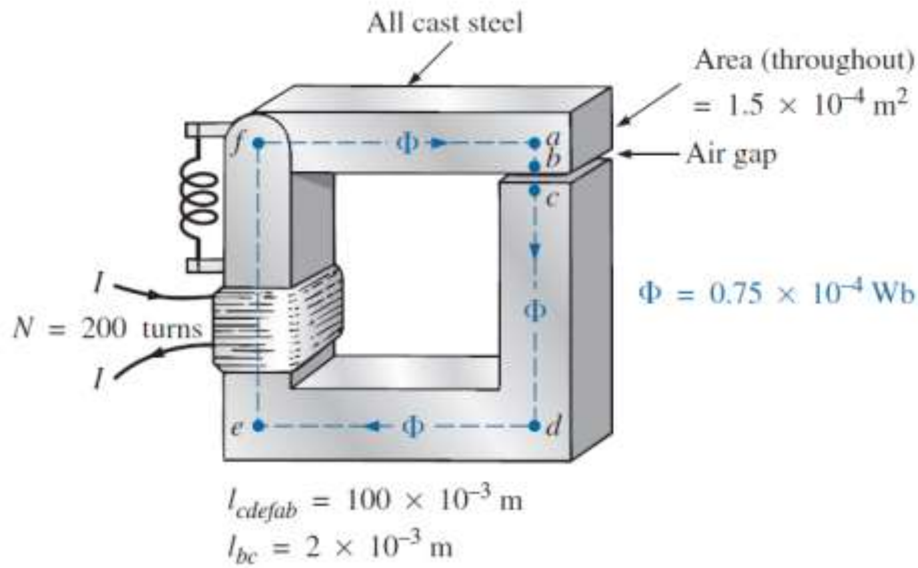
For most practical applications, the permeability of air is taken to be equal to that of free space. The magnetizing force of the air gap is then determined by

$$\boxed{H_g = \frac{B_g}{\mu_o}}$$

$$H_g = \frac{B_g}{\mu_o} = \frac{B_g}{4\pi \times 10^{-7}}$$

$$\boxed{H_g = (7.96 \times 10^5)B_g} \qquad \text{(At/m)}$$

Ex.(1) Find the value of *I* required to establish a magnetic flux of $\Phi = 0.75 \times 10^{-14}$ Wb in the series magnetic circuit in Fig. 16



Fig(16)

Sol. An equivalent magnetic circuit and its electric circuit analogy are shown in Fig. 17 (a & b) :

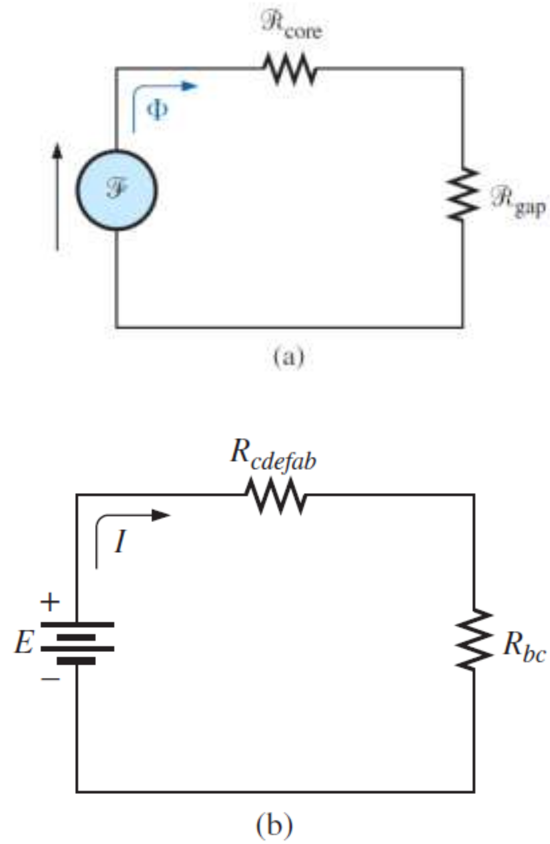


Fig.(17) *(a) Magnetic circuit equivalent and (b) electric circuit*

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{0.75 \times 10^{-4}\ \text{Wb}}{1.5 \times 10^{-4}\ \text{m}^2} = 0.5\ \text{T}$$

where, for our purposes,

$$\Phi_g = \Phi_{core}$$

and $$A_g = A_{core}$$

From the *B-H* curves in Fig(8-a)

$$H\ (\text{cast steel}) \cong 280\ \text{At/m}$$

$$H_g = (7.96 \times 10^5)B_g = (7.96 \times 10^5)(0.5\ \text{T}) = 3.98 \times 10^5\ \text{At/m}$$

The mmf drops are

$$H_{core}l_{core} = (280\ \text{At/m})(100 \times 10^{-3}\ \text{m}) = 28\ \text{At}$$
$$H_g l_g = (3.98 \times 10^5\ \text{At/m})(2 \times 10^{-3}\ \text{m}) = 796\ \text{At}$$

Applying Ampère's circuital law,

$$NI = H_{core}l_{core} + H_g l_g$$
$$= 28\ \text{At} + 796\ \text{At}$$
$$(200\ \text{t})I = 824\ \text{At}$$
$$I = \mathbf{4.12\ A}$$

Faraday's law of electromagnetic induction,

is one of the most important in this field because it enables us to establish ac and dc voltages with a generator. If we move a conductor through a magnetic field so that it cuts magnetic lines of flux as shown in Fig. (18) a voltage is induced across the conductor that can be measured with a sensitive voltmeter. the faster you move the conductor through the magnetic flux, the greater the induced voltage. The same effect can be produced if you hold the conductor still and move the magnetic field across the conductor. Note that the direction in which you move the conductor through the field determines the polarity of the induced voltage. Also, if you move the conductor through the field at right angles to the magnetic flux, you generate the maximum induced voltage. Moving the conductor parallel with the magnetic flux lines results in an induced voltage of zero volts since magnetic lines of flux are not crossed.

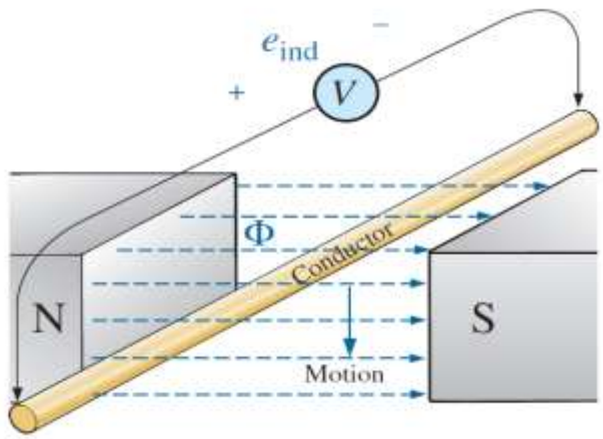


Fig.(18)

Faraday's law:

$$e = N\frac{d\phi}{dt} \qquad \text{(volts, V)}$$

Lenz's law

This important phenomenon can now be applied to the inductor in Fig.(19) , the magnetic flux linking the coil of *N* turns with a current *I* has the distribution shown in Fig. (19). If the current through the coil increases in magnitude, the flux linking the coil also increases. The coil in the vicinity of a changing magnetic flux will have a voltage induced across it. The result is that a voltage is induced across the coil in Fig. (19) due to the *change in current through the coil.*

It is very important to note in Fig. (19) that the polarity of the induced voltage across the coil is such that it opposes the increasing level of current in the coil. In other words, the changing current through the coil induces a voltage across the coil that is opposing the applied voltage that establishes the increase in current in the first place. The quicker the change in current through the coil, the greater the opposing induced voltage to squelch the attempt of the current to increase in magnitude.

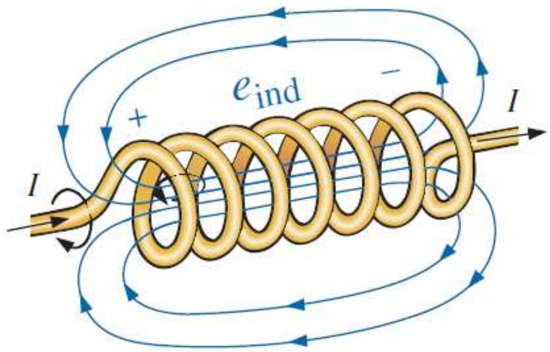


Fig.(19)

<u>Lenz's law</u>

an induced effect is always such as to oppose the cause that produced it.

<u>Self inductance</u>

inductors are designed to set up a strong magnetic field linking the unit, whereas capacitors are designed to set up a strong electric field between the plates.

Inductance is measured in **henries (H).**

$$\boxed{L = \frac{\mu N^2 A}{l}}$$

μ = permeability (Wb/A · m)
N = number of turns (t)
A = m^2
l = m
L = henries (H)

$$\mu = \mu_r \mu_o$$

$$L = \frac{\mu_r \mu_o N^2 A}{l}$$

or $$L = 4\pi \times 10^{-7} \frac{\mu_r N^2 A}{l}$$ (henries, H)

If we break out the relative permeability as follows:

$$L = \mu_r \left(\frac{\mu_o N^2 A}{l} \right)$$

we obtain the following useful equation:

$$L = \mu_r L_o$$

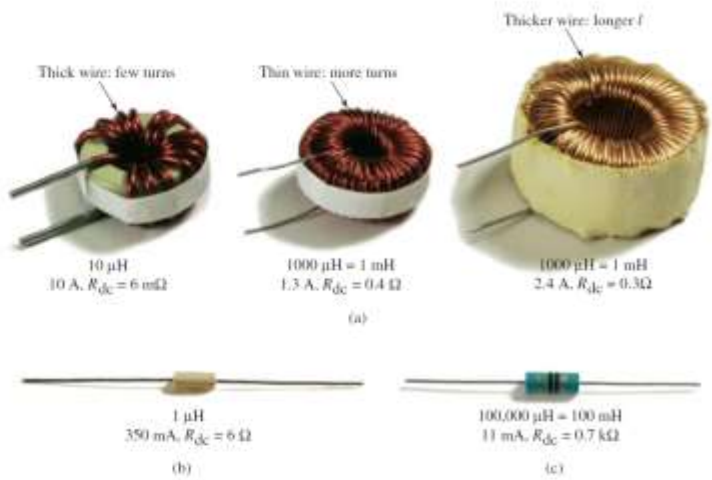
Note:

☺ ***The inductance of an inductor with a ferromagnetic core is μ_r times the inductance obtained with an air core.***

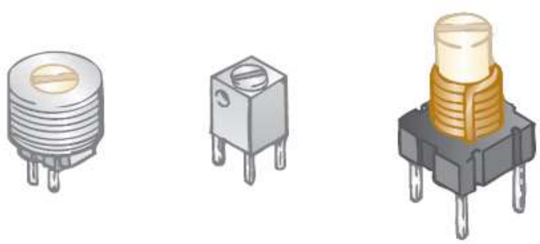
$$L = \mu_r L_o$$

Types of Inductors

Fixed-type inductors come in all shapes and sizes



Variable A number of variable inductors ,the inductance is changed by turning the slot at the end of the core to move it in and out of the unit.



INDUCED VOLTAGE

a voltage will be induced across the coil as determined by **Faraday's law:**

$$e = N\frac{d\phi}{dt} \qquad \text{(volts, V)}$$

The greater the number of turns or the faster the coil is moved through the magnetic flux pattern, the greater the induced voltage.

the induced voltage across an inductor

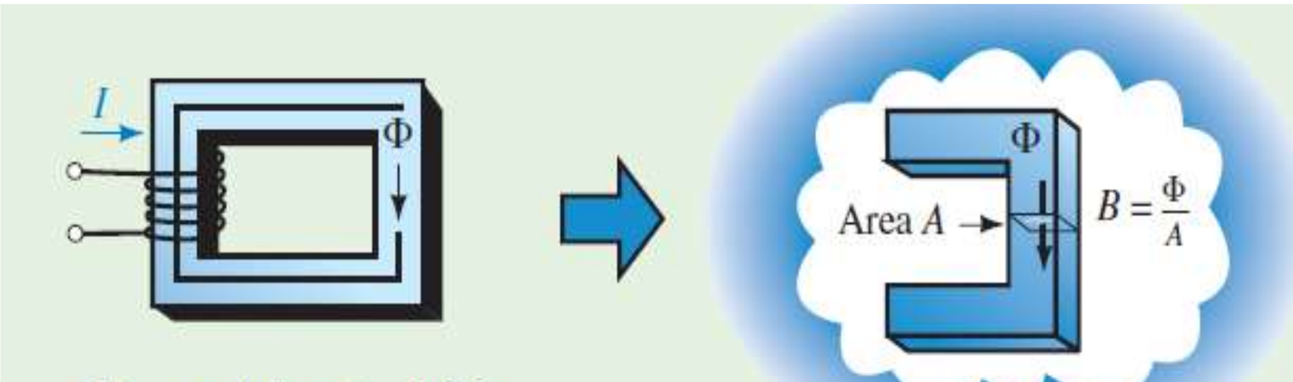
$$v_L = L\frac{di_L}{dt} \quad \text{(volts, V)}$$

the larger the inductance and/or the more rapid the change in current through a coil, the larger will be the induced voltage across the coil.

ENERGY STORED BY AN INDUCTOR

$$W_{\text{stored}} = \frac{1}{2}LI_m^2 \quad \text{(joules, J)}$$

<u>Ex.</u> If the core of Figure below is cast iron and $\phi = 0.1 \times 10^{-3}$Wb, what is the coil current?



Mean path length = 0.25 m
$N = 500$ turns
$A = 0.2 \times 10^{-3}\ m^2$

Solution Following the four basic steps:

1. The flux density is

$$B = \frac{\Phi}{A} = \frac{0.1 \times 10^{-3}}{0.2 \times 10^{-3}} = 0.5\ \text{T}$$

2. From the *B*-*H* curve (cast iron), H = 1550 At/m.

3. Apply Ampere's law. There is only one coil and one core section. Length = 0.25 m. Thus,

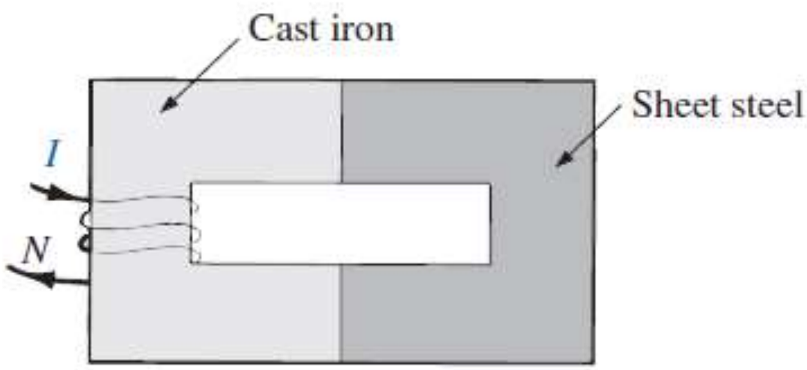
$$H\ell = 1550 \times 0.25 = 388\ \text{At} = NI$$

4. Solve for *I*:

$$I = H\ell/N = 388/500 = 0.78\ \text{amps}$$

Exercise:

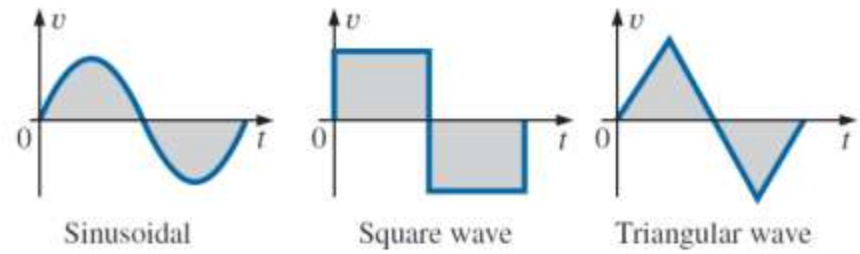
Find the current necessary to establish a flux of $\Phi = 3 \times 10^{-4}$ W_b in the series magnetic circuit in Fig. below:



$l_{iron\ core} = l_{steel\ core} = 0.3$ m
Area (throughout) $= 5 \times 10^{-4}$ m^2
$N = 100$ turns

Exercise: Find the reluctance of a magnetic circuit if a magnetic flux $\Phi = 4.2 \times 10^{-4}$ W_b is established by an impressed mmf of 400 At.

Alternating Waveforms



- Important parameters for a sinusoidal voltage fig.(1).

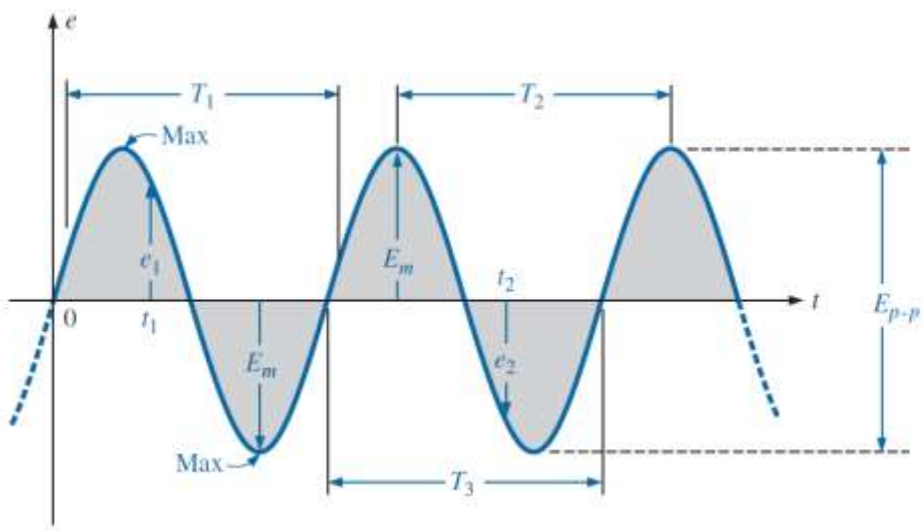


Fig (1) important parameters for a sinusoidal voltage

a- Waveform plotted as a function of some variable such as time (as above), position, degrees, radians, temperature, and so on.

b- The magnitude of a waveform at any instant of time; denoted by lower case letters (e1, e2 in Fig. 1).

c- Peak amplitude: The maximum value of a waveform as measured from its average, or mean, value, denoted by uppercase letters [such as Em (Fig. 1)

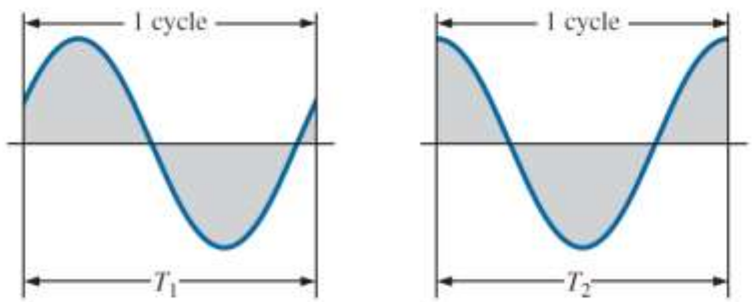
d- Peak value: The maximum instantaneous value of a function as measured from the zero volt level.

e- Peak-to-peak value: Denoted by (E p-p or V p-p) (as shown in Fig. 1), the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks

f- Periodic waveform: A waveform that continually repeats itself after the same time interval. The waveform in (Fig. 1) is a periodic waveform.

g- Period (T): The time of a periodic waveform

h- Cycle: The portion of a waveform contained in one period of time the cycles within T1, T2, in Fig. (1) May appear different in Fig. (2), but they are all bounded by one period of time and therefore satisfy the definition of a cycle.



Fig(2) Defining the cycle and period of a sinusoidal waveform

i- Frequency (f): The number of cycles that occur in 1 s.
The unit of measure for frequency is the hertz (Hz), where

1 hertz (Hz) = 1 cycle per second (cps)

$$f = \frac{1}{T} \qquad f = \text{Hz}, \quad T = \text{seconds (s)} \qquad \longrightarrow 1$$

$$T = \frac{1}{f} \qquad \longrightarrow 2$$

Ex.1) For the sinusoidal waveform in Fig. (3)

a. What is the peak value?
b. What is the instantaneous value at 0.3 s and 0.6 s?
c. What is the peak-to-peak value of the waveform?
d. What is the period of the waveform?
e. How many cycles are shown?
f. What is the frequency of the waveform?

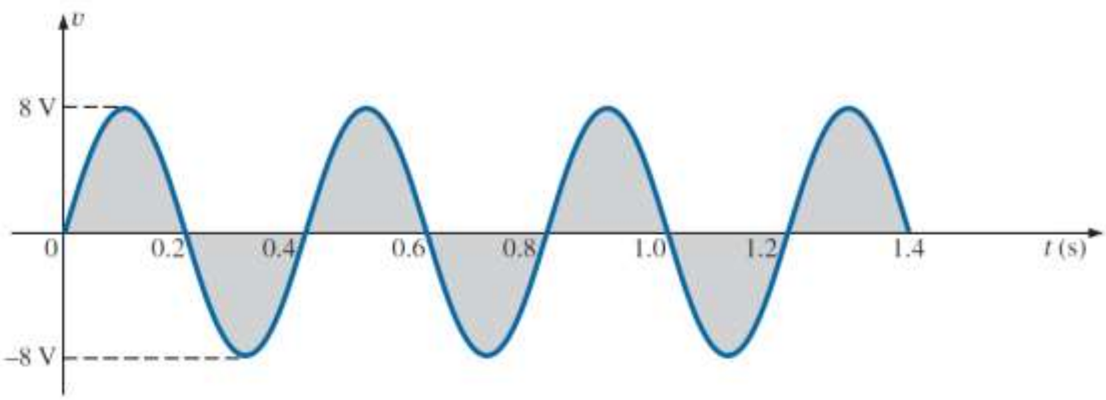


Fig.(3)

Solutions:

a. **8 V.**
b. At 0.3 s, **−8 V;** at 0.6 s, **0 V.**
c. **16 V.**
d. **0.4 s.**
e. **3.5 cycles.**
f. **2.5 cps,** or **2.5 Hz.**

(Example 2)Find the period of periodic waveform with a frequency of:

a. 60 Hz.
b. 1000 Hz.

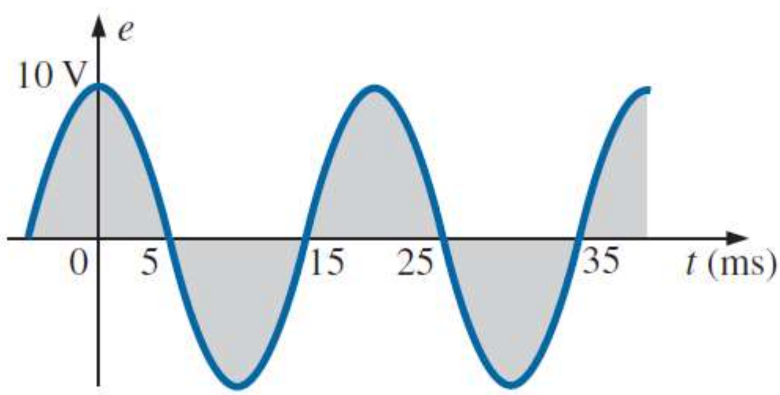
Solutions:

a. $T = \frac{1}{f} = \frac{1}{60\ \text{Hz}} \cong 0.01667\ \text{s}$ or **16.67 ms**

(a recurring value since 60 Hz is so prevalent)

b. $T = \frac{1}{f} = \frac{1}{1000\ \text{Hz}} = 10^{-3}\ \text{s} = \mathbf{1\ ms}$

Example 3) Determine the frequency of the waveform in Fig. (4)



Fig(4)

Solution: From the figure, $T = (25\ \text{ms} - 5\ \text{ms})$ or $(35\ \text{ms} - 15\ \text{ms}) = 20\ \text{ms}$, and

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}\ \text{s}} = \mathbf{50\ Hz}$$

- Polarities and Direction(fig.5)

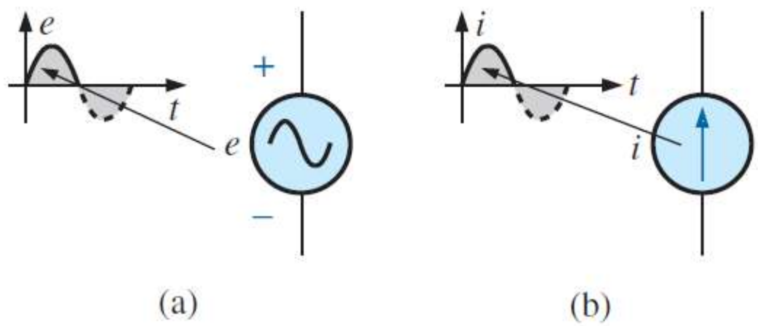


Fig.(5) (a) Sinusoidal ac voltage sources (b) sinusoidal current sources.

THE SINUSOIDAL WAVEFORM

The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C elements.(fig.6)

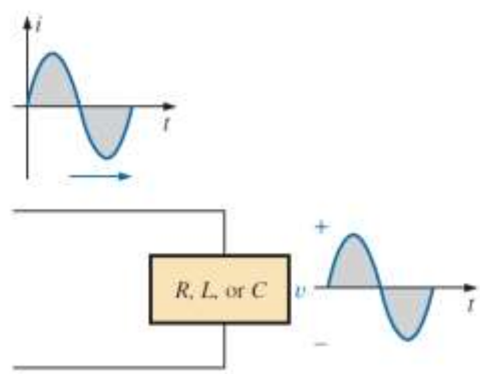


Fig.(6)

The sine wave is the only alternating waveform whose shape is not altered by the response characteristics of a pure resistor, inductor, or capacitor.

The unit of measurement for the horizontal axis can be time (as appearing in the figures thus far), degrees, or radians. The term radian can be defined as follows: If we mark off a portion of the circumference of a circle by a length equal to the radius of the circle, as shown in Fig. (7), the angle resulting is called 1 radian. The result is:

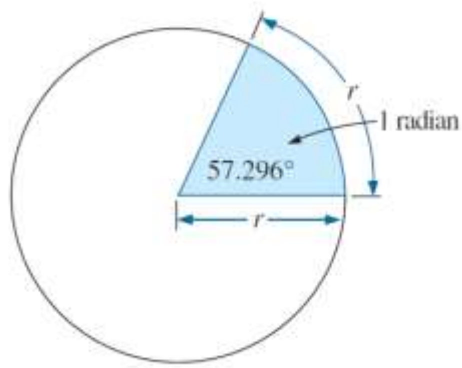


Fig (7)

$$1 \text{ rad} = 57.296° \cong 57.3°$$

One full circle has 2π radians

$$2\pi \text{ rad} = 360°$$

The quantity π is the ratio of the circumference of a circle to its diameter.
Although the approximation π ≈ 3.14

The conversions equations between the two (radian and degrees) are the following:

$$\text{Radians} = \left(\frac{\pi}{180°}\right) \times (\text{degrees}) \quad (3)$$

$$\text{Degrees} = \left(\frac{180°}{\pi}\right) \times (\text{radians}) \quad (4)$$

Applying these equations, we find

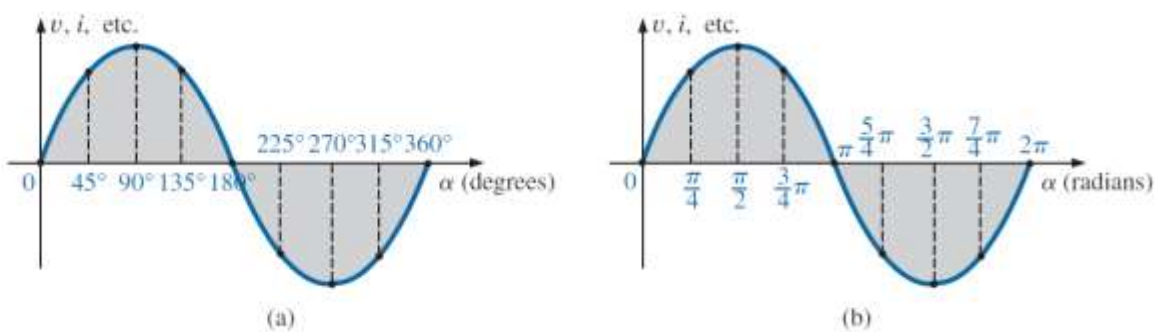
$$\mathbf{90^\circ}\text{:}\quad \text{Radians} = \frac{\pi}{180^\circ}(90^\circ) = \frac{\pi}{\mathbf{2}}\ \mathbf{rad}$$

$$\mathbf{30^\circ}\text{:}\quad \text{Radians} = \frac{\pi}{180^\circ}(30^\circ) = \frac{\pi}{\mathbf{6}}\ \mathbf{rad}$$

$$\frac{\pi}{\mathbf{3}}\ \mathbf{rad}\text{:}\quad \text{Degrees} = \frac{180^\circ}{\pi}\left(\frac{\pi}{3}\right) = \mathbf{60^\circ}$$

$$\frac{\mathbf{3}\pi}{\mathbf{2}}\ \mathbf{rad}\text{:}\quad \text{Degrees} = \frac{180^\circ}{\pi}\left(\frac{3\pi}{2}\right) = \mathbf{270^\circ}$$

See fig.(8)



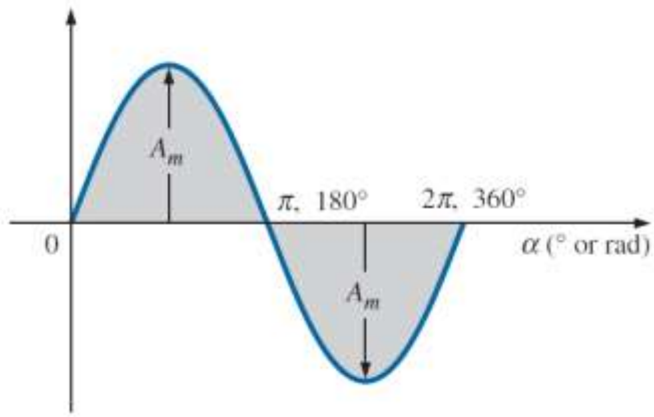
(a) degrees (b) radians.

Fig(8) Plotting a sine wave

1- GENERAL FORMAT FOR THE SINUSOIDAL VOLTAGE OR CURRENT

The basic mathematical format for the sinusoidal waveform is:

$$A_m \sin \alpha$$ ------------(5



Fig(9) Basic sinusoidal function.

where Am is the peak value of the waveform and a is the unit of measure for the horizontal axis, as shown in Fig. (9)
the general format of a sine wave can also be written:

$$A_m \sin \omega t$$ ----------(6

with ωt as the horizontal unit of measure
For electrical quantities such as current and voltage, the general format is:

$$i = I_m \sin \omega t = I_m \sin \alpha$$

$$e = E_m \sin \omega t = E_m \sin \alpha$$

------------(7

Where the capital letters with the subscript (m) represent the amplitude, and the lowercase letters(i) and(e)represent the instantaneous value of current and voltage, respectively, at any time(t)

Example 4). Given e=5 sin α, determine e at α =40° and α=0.8π.

Solution: For $\alpha = 40°$,

$$e = 5 \sin 40° = 5(0.6428) = \mathbf{3.21\ V}$$

For $\alpha = 0.8\pi$,

$$\alpha\,(°) = \frac{180°}{\pi}(0.8\pi) = 144°$$

and $$e = 5 \sin 144° = 5(0.5878) = \mathbf{2.94\ V}$$

$e = E_m \sin \alpha$

$$\sin \alpha = \frac{e}{E_m}$$

which can be written

$$\alpha = \sin^{-1}\frac{e}{E_m} \quad \text{------------------------------------ (8}$$

Similarly, for a particular current level,

$$\alpha = \sin^{-1}\frac{i}{I_m} \quad \text{------------------------------------ (9}$$

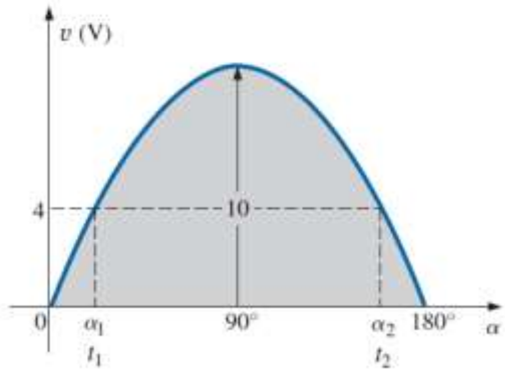
Example 5)

a. Determine the angle at which the magnitude of the sinusoidal function:
v = 10 sin 377t is 4 V.
The magnitude is attained.

Solutions:

$$\alpha_1 = \sin^{-1}\frac{v}{E_m} = \sin^{-1}\frac{4\ V}{10\ V} = \sin^{-1} 0.4 = \mathbf{23.58°}$$

From the fig.(10)



Fig(10)

the magnitude of 4 V (positive) will be attained at two points between 0° and 180°. The second intersection is determined by:

$$\alpha_2 = 180° - 23.578° = \mathbf{156.42°}$$

Note) keep in mind that Eqs. (8) and (9) will provide an angle with a magnitude between 0° and 90°.

$$\alpha = \omega t, \text{ and so } t = \alpha/\omega.$$

(α) must be in radians.

$$\alpha\,(\text{rad}) = \frac{\pi}{180^\circ}(23.578^\circ) = 0.412 \text{ rad}$$

and $$t_1 = \frac{\alpha}{\omega} = \frac{0.412 \text{ rad}}{377 \text{ rad/s}} = \mathbf{1.09\ ms}$$

For the second intersection,

$$\alpha\,(\text{rad}) = \frac{\pi}{180^\circ}(156.422^\circ) = 2.73 \text{ rad}$$

$$t_2 = \frac{\alpha}{\omega} = \frac{2.73 \text{ rad}}{377 \text{ rad/s}} = \mathbf{7.24\ ms}$$

Example 6) Given $i = 6 \times 10^{-3} \sin 1000t$, determine i at t =2ms

Solution:

$$\alpha = \omega t = 1000t = (1000 \text{ rad/s})(2 \times 10^{-3} \text{ s}) = 2 \text{ rad}$$

$$\alpha\,(^\circ) = \frac{180^\circ}{\pi \text{ rad}}(2 \text{ rad}) = 114.59^\circ$$

$$i = (6 \times 10^{-3})(\sin 114.59^\circ) = (6 \text{ mA})(0.9093) = \mathbf{5.46\ mA}$$

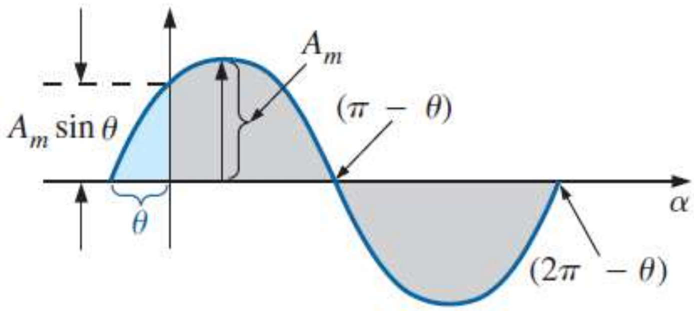
2- PHASE RELATIONS

If the waveform is shifted to the right or left of Θ°, the expression becomes

$$A_m \sin(\omega t \pm \theta)$$

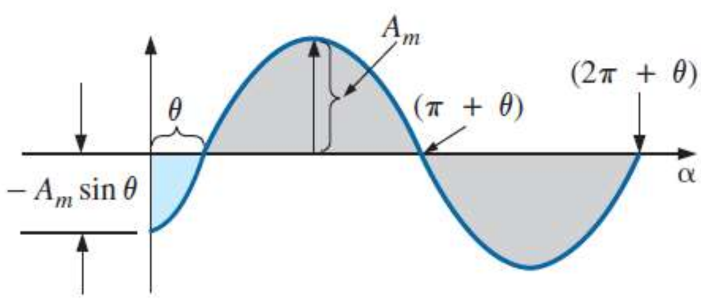
Here $\boldsymbol{\Theta}$ is the angle in degrees or radians that the waveform has been shifted. If the waveform passes through the horizontal axis with a positive going (increasing with time) slope before 0°, as shown in Fig. (11), the expression is:

$$A_m \sin(\omega t + \theta)$$



Fig(11)

At ωt = a = 0°, the magnitude is determined by Am sin Θ. If the waveform passes through the horizontal axis with a positive-going slope after 0°, as shown in Fig. (12), the expression is:



Fig(12)

$$A_m \sin(\omega t - \theta)$$

If the waveform crosses the horizontal axis with a positive-going slope 90° (π/2), as shown in Fig. (13), it is called a cosine wave; that is,

$$\sin(\omega t + 90°) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

or

$$\sin \omega t = \cos(\omega t - 90°) = \cos\left(\omega t - \frac{\pi}{2}\right)$$

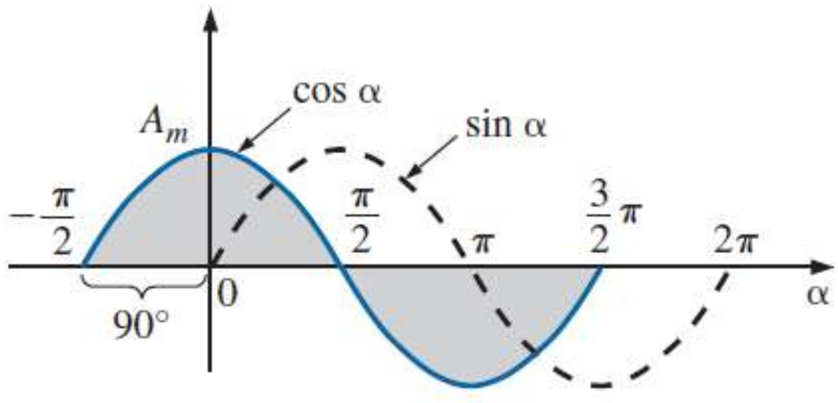


Fig.(13) Phase relationship between a sine wave and a cosine wave.

The terms **leading** and **lagging** are used to indicate the relationship between two sinusoidal waveforms of the same frequency plotted on the same set of axes. In Fig.(14), the cosine curve is said to lead the sine curve by 90°, and the sine curve is said to lag the cosine curve by 90°. ***The 90° is referred to as the phase angle between the two waveforms.***

The **phase relationship** between two waveforms indicates which one leads or lags the other, and by how many degrees or radians.

3- relationship between specific sine and cosine functions

$$\cos\alpha = \sin(\alpha + 90^\circ)$$
$$\sin\alpha = \cos(\alpha - 90^\circ)$$
$$-\sin\alpha = \sin(\alpha \pm 180^\circ)$$
$$-\cos\alpha = \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ)$$
etc.

In addition, note that

$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$

FOR THE SINUSOIDAL VOLTAGE

$$e = -E_m \sin \omega t$$

Become

$$e = E_m(-\sin \omega t)$$

Since $-\sin \omega t = \sin(\omega t \pm 180^\circ)$

the expression can also be written

$$e = E_m \sin(\omega t \pm 180^\circ)$$

revealing that a negative sign can be replaced by a 180° change in phase angle (+ or −); that is,

$$e = -E_m \sin \omega t = E_m \sin(\omega t + 180^\circ) = E_m \sin(\omega t - 180^\circ)$$

EXAMPLE 7) What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10 \sin(\omega t + 30^\circ)$
   $i = 5 \sin(\omega t + 70^\circ)$
b. $i = 15 \sin(\omega t + 60^\circ)$
   $v = 10 \sin(\omega t - 20^\circ)$
c. $i = 2 \cos(\omega t + 10^\circ)$
   $v = 3 \sin(\omega t - 10^\circ)$
d. $i = -\sin(\omega t + 30^\circ)$
   $v = 2 \sin(\omega t + 10^\circ)$
e. $i = -2 \cos(\omega t - 60^\circ)$
   $v = 3 \sin(\omega t - 150^\circ)$

<u>Solution</u>:

a)

***i* leads *v* by 40º, or *v* lags *i* by 40º.**

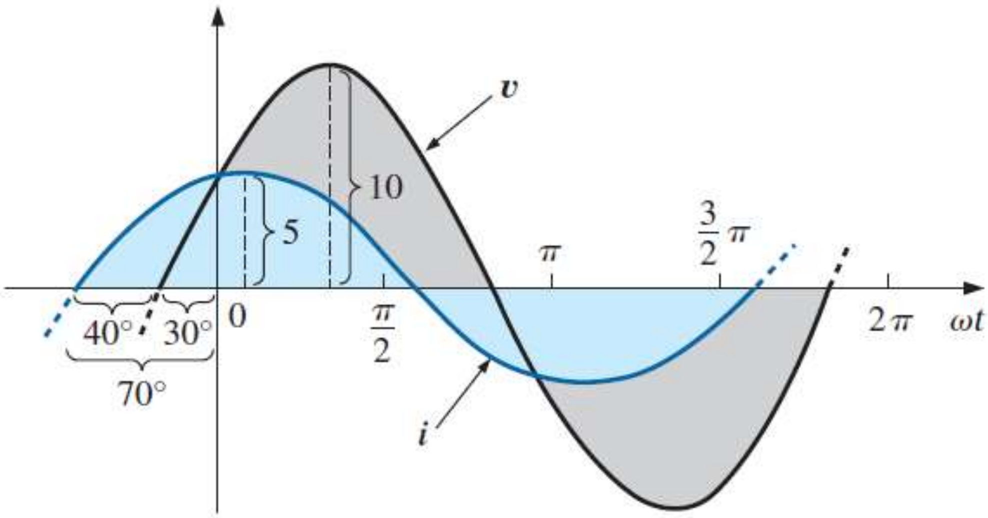


Fig.(14)

b) **i leads** v **by 80º, or** v **lags i by 80º.**

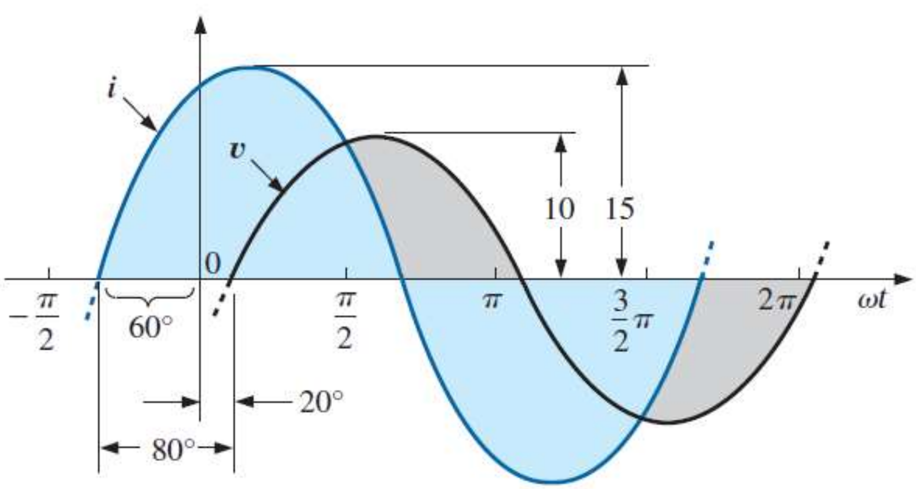


Fig.(15)

c)

$$i = 2\cos(\omega t + 10°) = 2\sin(\omega t + 10° + 90°)$$
$$= 2\sin(\omega t + 100°)$$

***i* leads *v* by 110º, or *v* lags *i* by 110º.**

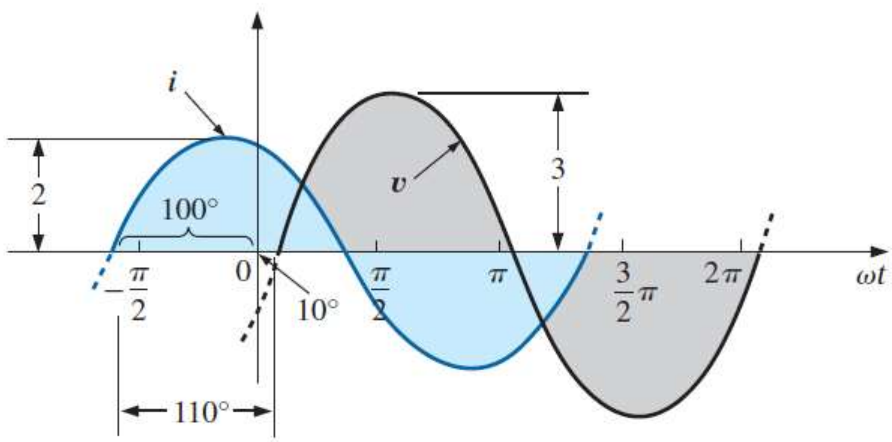


Fig.(16)

d.

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° \overset{\text{Note}}{-} 180°)$$
$$= \sin(\omega t - 150°)$$

***v* leads *i* by 160º, or *i* lags *v* by 160º.**

Or using

$$-\sin(\omega t + 30°) = \sin(\omega t + 30° \overset{\text{Note}}{+} 180°)$$
$$= \sin(\omega t + 210°)$$

***i* leads *v* by 200º, or *v* lags *i* by 200º.**

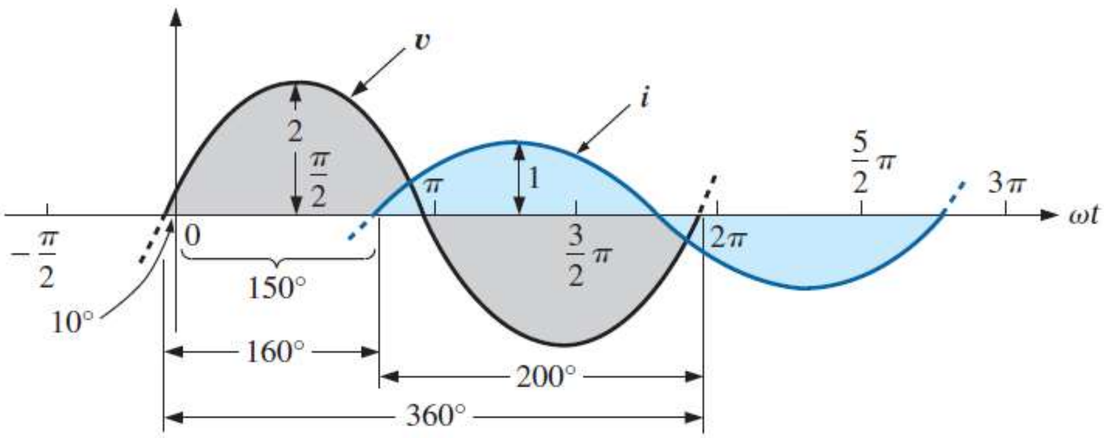


Fig.(17)

e.

By choice

$$i = -2\cos(\omega t - 60^\circ) = 2\cos(\omega t - 60^\circ - 180^\circ)$$
$$= 2\cos(\omega t - 240^\circ)$$

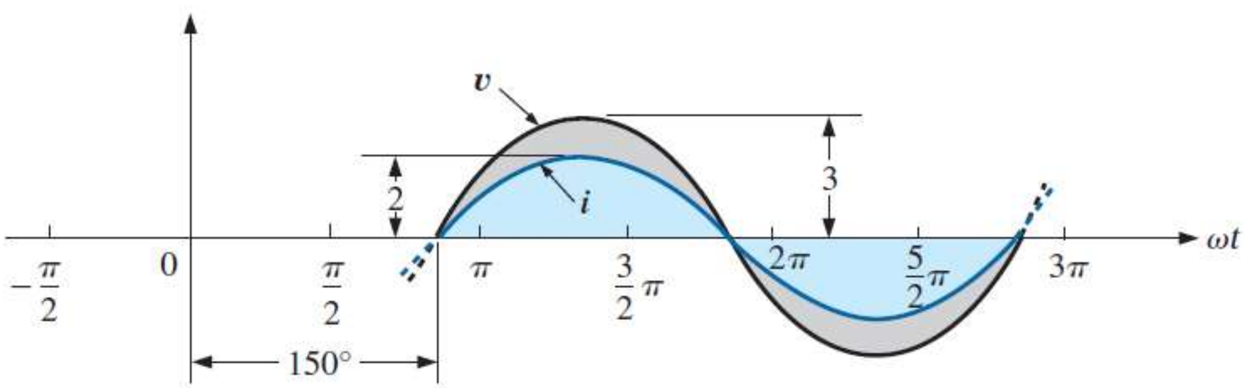


Fig.(18)

However, $\cos \alpha = \sin(\alpha + 90°)$

so that
$$2\cos(\omega t - 240°) = 2\sin(\omega t - 240° + 90°)$$
$$= 2\sin(\omega t - 150°)$$

***v* and *i* are in phase.**

4- AVERAGE VALUE

$$\text{Average speed} = \frac{\text{area under curve}}{\text{length of curve}}$$

if we let G denote the average value, The algebraic sum of the areas become:

$$G\ (\text{average value}) = \frac{\text{algebraic sum of areas}}{\text{length of curve}}$$

Example(8)

Determine the average value of the waveforms in fig.(19)

Fig.(1-9)

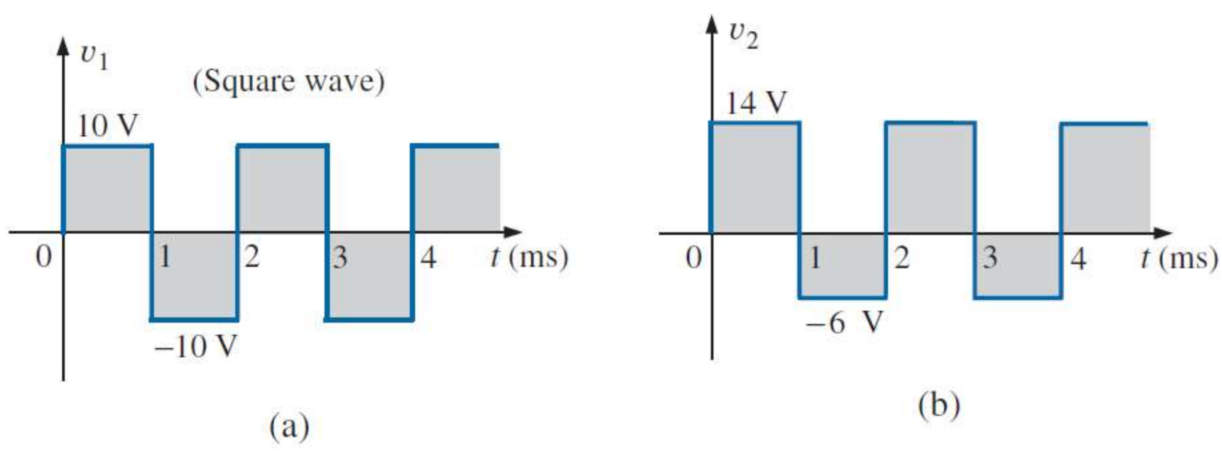


Fig.(19)

Solution:

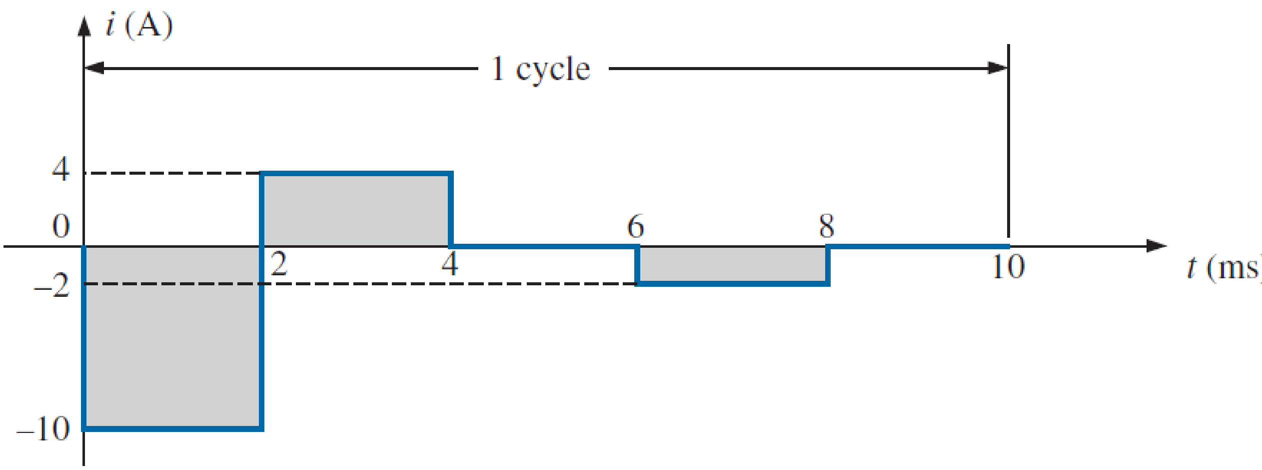
a)

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

b)

$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}} = \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

Example (9):Find the average values of the following waveform over one full cycle in fig.(20):



Fig(20)

Solution:

$$G = \frac{-(10\text{ V})(2\text{ ms}) + (4\text{ V})(2\text{ ms}) - (2\text{ V})(2\text{ ms})}{10\text{ ms}}$$

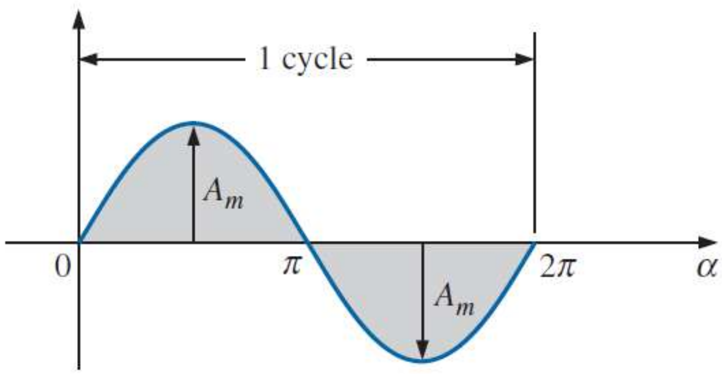
$$= \frac{-20\text{ V} + 8\text{ V} - 4\text{ V}}{10} = -\frac{16\text{ V}}{10} = \mathbf{-1.6\ V}$$

For sine wave:

the area of the positive (or negative) pulse of a sine wave is $2A_m$.

$$\text{Area} = 2A_m$$

Example (10) Determine the average value of the sinusoidal waveform in Fig. (21)



Fig(21)

Solution:

$$G = \frac{+2A_m - 2A_m}{2\pi} = \mathbf{0\ V}$$

The average value of a pure sinusoidal waveform over one full cycle is zero.

Example (11): Determine the average value of the waveform in Fig. (22):

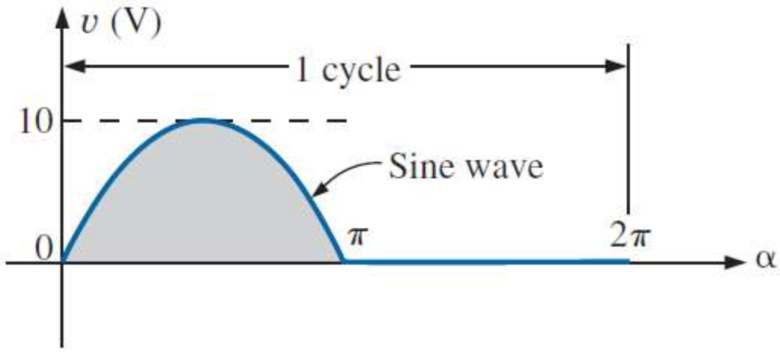


fig.(22)

Solution:

$$G = \frac{2A_m + 0}{2\pi} = \frac{2(10\text{ V})}{2\pi} \cong \mathbf{3.18\ V}$$

5- EFFECTIVE (rms) VALUES

the equivalent dc value of a sinusoidal current or voltage is(1/ √2) or (0.707) of its peak value.
The equivalent dc value is called the(**root-mean-square) (rms) value** or **effective value** of the sinusoidal quantity.

$$I_{rms} = \frac{1}{\sqrt{2}} I_m = 0.707 I_m$$

$$E_{rms} = \frac{1}{\sqrt{2}} E_m = 0.707 E_m$$

Similarly,

$$I_m = \sqrt{2} I_{rms} = 1.414 I_{rms}$$

$$E_m = \sqrt{2} E_{rms} = 1.414 E_{rms}$$

Example (12) Find the rms values of the sinusoidal waveform in each part in Fig(23):

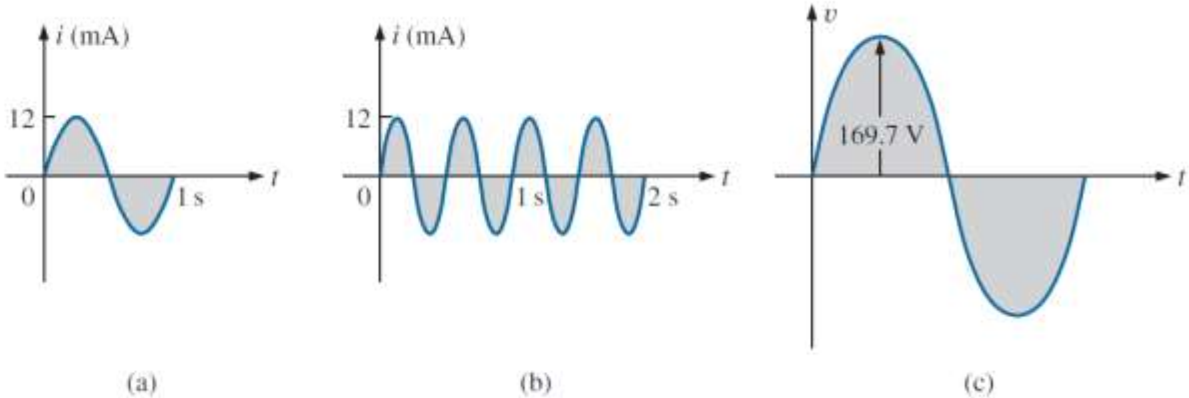


Fig.(23)

Solution: For part (a), $I_{rms} = 0.707(12 \times 10^{-3}\ A) =$ **8.48 mA.** For part (b), again $I_{rms} =$ **8.48 mA.** Note that frequency did not change the effective value in (b) compared to (a). For part (c), $V_{rms} = 0.707(169.73\ V) \cong$ **120 V,** the same as available from a home outlet.

Example (13) The 120 V dc source in Fig.[(24)(a)} delivers 3.6 W to the load. Determine the peak value of the applied voltage (*Em*) and the current (*Im*) if the ac source [Fig. (24)(b)] is to deliver the same power to the load.

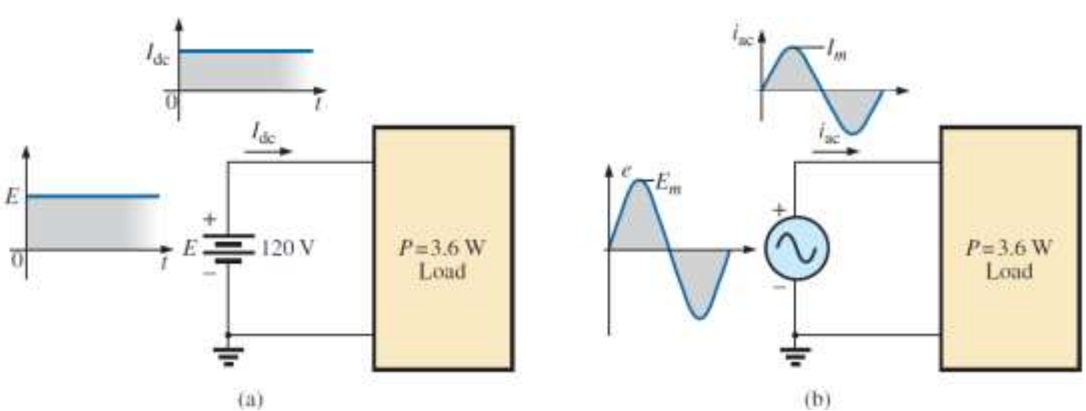


Fig.24

Solution:

$$P_{dc} = V_{dc} I_{dc}$$

and $$I_{dc} = \frac{P_{dc}}{V_{dc}} = \frac{3.6\text{ W}}{120\text{ V}} = 30\text{ mA}$$

$$I_m = \sqrt{2} I_{dc} = (1.414)(30\text{ mA}) = \mathbf{42.42\text{ mA}}$$

$$E_m = \sqrt{2} E_{dc} = (1.414)(120\text{ V}) = \mathbf{169.68\text{ V}}$$

Example(14): Find the rms value of the waveform in Fig.(25)

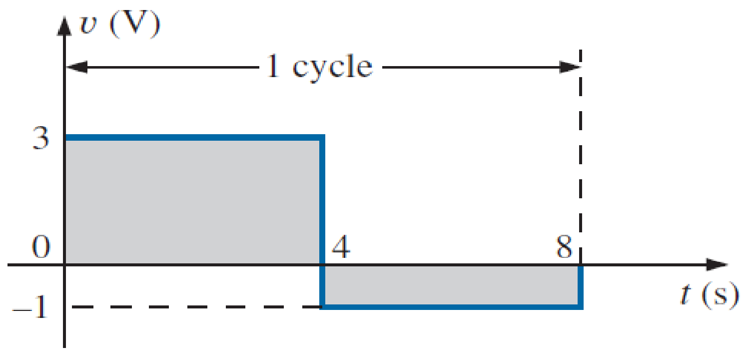


Fig.25

Solution: V^2 in fig.(26)

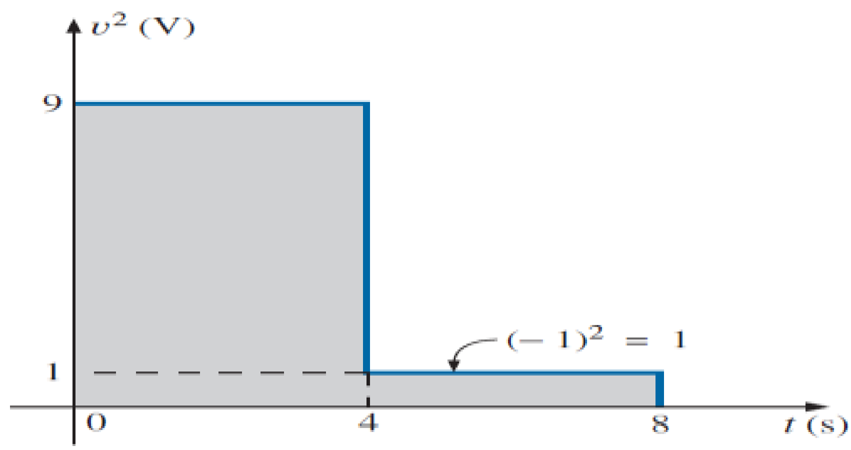


Fig.26

$$V_{rms} = \sqrt{\frac{(9)(4) + (1)(4)}{8}} = \sqrt{\frac{40}{8}} = \mathbf{2.24\ V}$$

Example(15) : Calculate the rms value of the voltage in Fig (27):

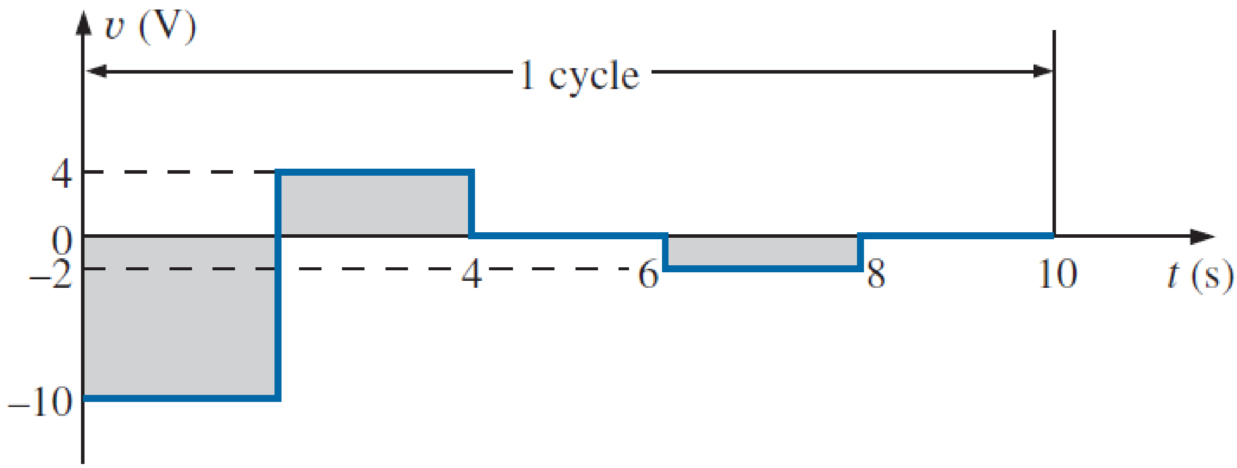


Fig.(27)

Solution :v2in fig.(28)

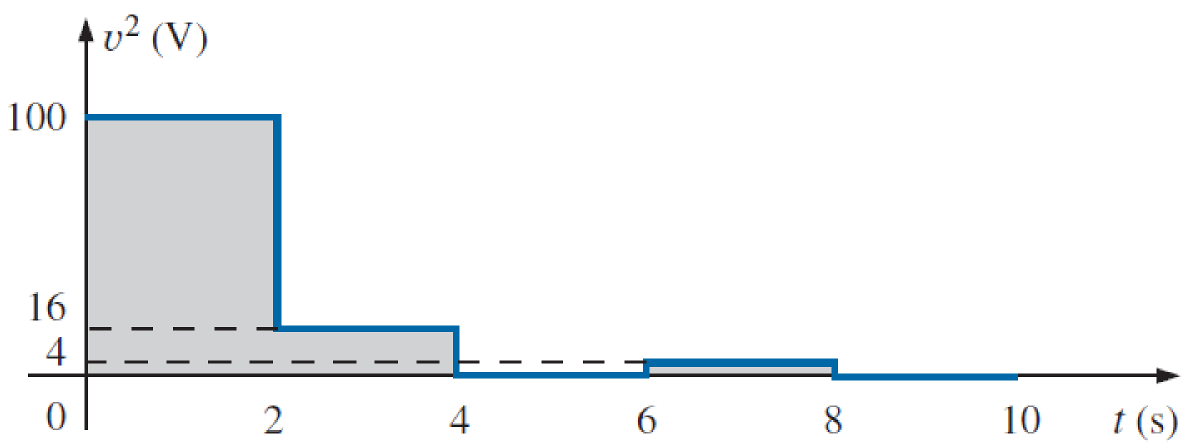


Fig.(28)

Example(16): Determine the average and (rms)values of the square wave in Fig. (29):

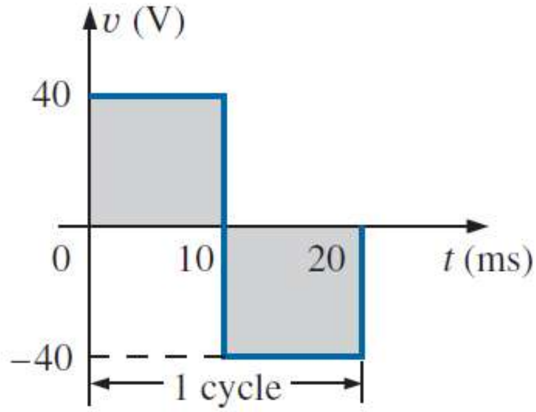


Fig.(29)

Solution: v^2in fig.(33)

$$V_{rms} = \sqrt{\frac{(1600)(10 \times 10^{-3}) + (1600)(10 \times 10^{-3})}{20 \times 10^{-3}}}$$

$$= \sqrt{\frac{(32{,}000 \times 10^{-3})}{20 \times 10^{-3}}} = \sqrt{1600} = \mathbf{40\ V}$$

(the maximum value of the waveform in Fig.(30)

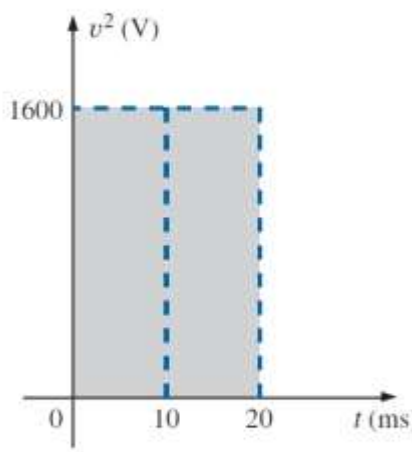
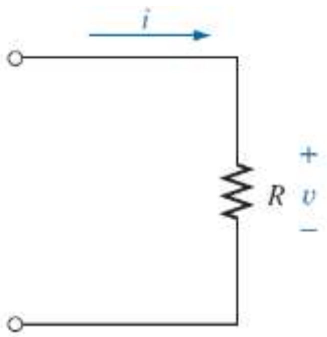


Fig.(30)

6- RESPONSE OF BASIC R, L, AND C ELEMENTS TO A SINUSOIDAL VOLTAGE OR CURRENT

a- **Resistor:** resistance is, for all practical purposes, unaffected by the frequency of the applied sinusoidal voltage or current.



Determining the sinusoidal response for a resistive element

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

$$I_m = \frac{V_m}{R}$$

$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

$$V_m = I_m R$$

A plot of v and i in Fig(1) reveals that

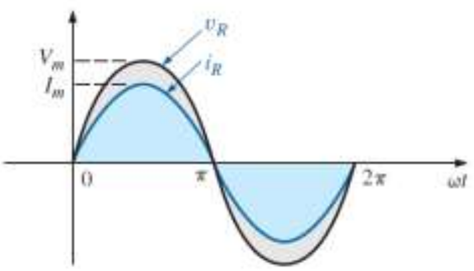
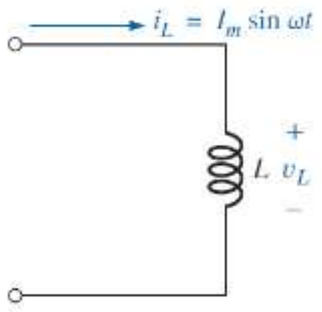


Fig.(1) The voltage and current of a resistive element are in phase.

for a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.

b- **Inductor:** *for an inductor, v_L leads i_L by 90°, or i_L lags v_L by 90°.*

in fig.(2):



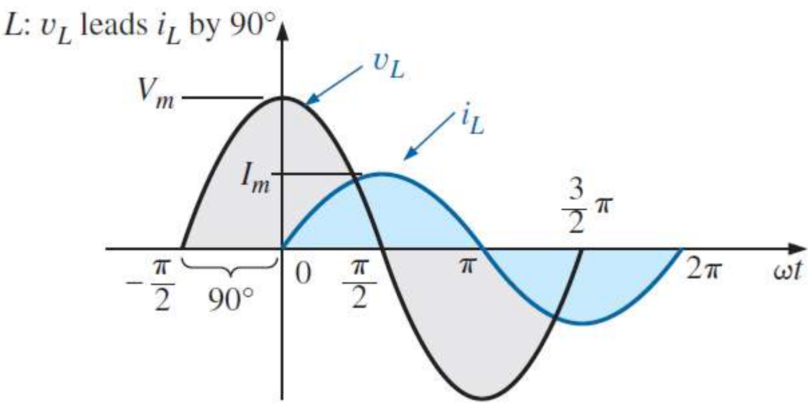


Fig.(2) For a pure inductor, the voltage across the coil leads the current through the coil by 90°.

$$i_L = I_m \sin(\omega t \pm \theta)$$
$$v_L = \omega L I_m \sin(\omega t \pm \theta + 90°)$$

The quantity ωL, called the **reactance** of an inductor, is symbolically represented by XL and is measured in ohms;

$$X_L = \omega L$$

In an Ohm's law format, its magnitude can be determined from:

$$X_L = \frac{V_m}{I_m} \quad \text{(ohms, } \Omega\text{)}$$

c- **Capacitor**

for a capacitor, i_C leads v_C by 90°, or v_C lags i_C by 90°.

In fig.(3):

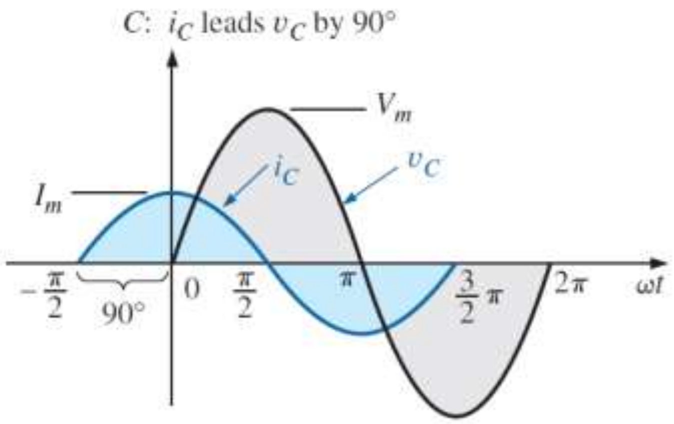


Fig.(3) The current of a purely capacitive element leads the voltage across the element by 90°.

$$v_C = V_m \sin(\omega t \pm \theta)$$
$$i_C = \omega C V_m \sin(\omega t \pm \theta + 90°)$$

$$\frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

The quantity (1/ωc) called the **reactance** of a capacitor, is symbolically represented by XC and is measured in ohms; that is:

$$X_C = \frac{1}{\omega C} \quad \text{(ohms, } \Omega\text{)}$$

In an Ohm's law format, its magnitude can be determined from:

$$X_C = \frac{V_m}{I_m} \qquad \text{(ohms, } \Omega)$$

☺ It is possible to determine whether a network with one or more elements is predominantly capacitive or inductive by noting the phase relationship between the input voltage and current.

Note: If the source current leads the applied voltage, the network is predominantly capacitive, and if the applied voltage leads the source current, it is predominantly inductive.

Example1: The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 **Ω**. Sketch the curves for *v* and *i*.

a. $v = 100 \sin 377t$
b. $v = 25 \sin(377t + 60°)$

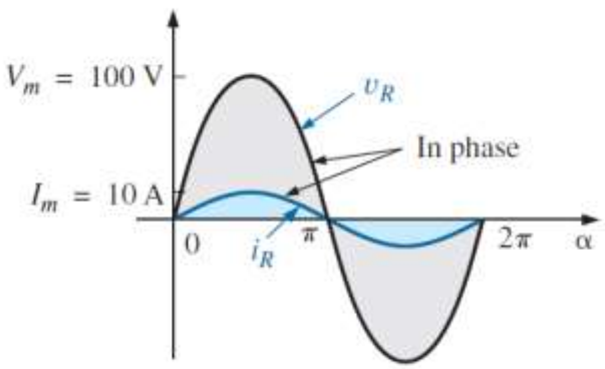
Solution: a)

$$I_m = \frac{V_m}{R} = \frac{100\text{ V}}{10\ \Omega} = 10\text{ A}$$

(v and i are in phase), resulting in

$$i = \mathbf{10 \sin 377}t$$

The curves are sketched in Fig. below:



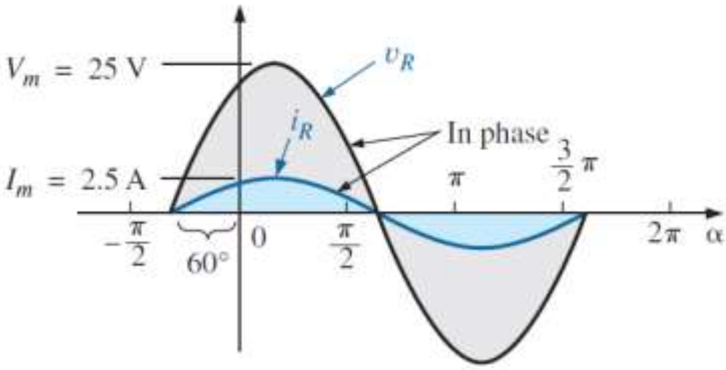
Solution: b)

$$I_m = \frac{V_m}{R} = \frac{25 \text{ V}}{10 \ \Omega} = 2.5 \text{ A}$$

(*v* and *i* are in phase), resulting in

$$i = \mathbf{2.5 \sin(377}t \mathbf{+ 60°)}$$

The curves are sketched in Fig. below:



<u>Example2</u>:
The current through a 5 Ω resistor is given. Find the sinusoidal expression for the voltage across the resistor for

i= 40 sin(377*t* +30°).

<u>Solution:</u>
Vm = *Im*.*R* = (40 A)(5 Ω) = 200 V
(*v* and *i* are in phase), resulting in ⟹ *v* = **200 sin(377*t* + 30°)**

<u>Example 3:</u> The current through a 0.1 H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the *v* and *i* curves.

a. $i = 10 \sin 377t$
b. $i = 7 \sin(377t - 70°)$

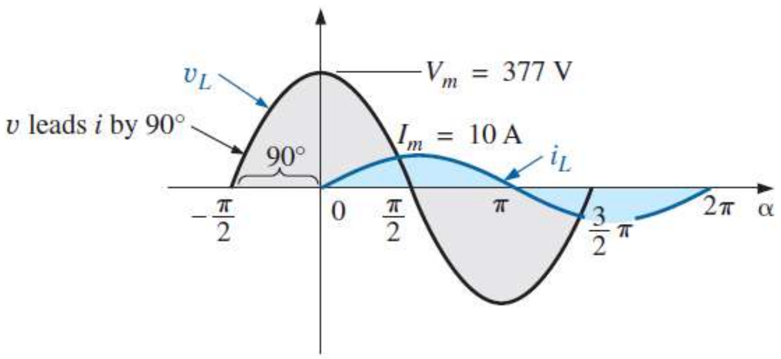
Solution:

$$X_L = \omega L = (377 \text{ rad/s})(0.1 \text{ H}) = 37.7\ \Omega$$
$$V_m = I_m X_L = (10 \text{ A})(37.7\ \Omega) = 377 \text{ V}$$

and we know that for a coil υ leads i by 90°. Therefore,

$$\boldsymbol{\upsilon = 377 \sin(377t + 90°)}$$

The curves are sketched below:



b. X_L remains at 37.7 Ω.

$V_m = I_m X_L = (7 \text{ A})(37.7\ \Omega) = 263.9 \text{ V}$

and we know that for a coil υ leads i by 90°. Therefore,

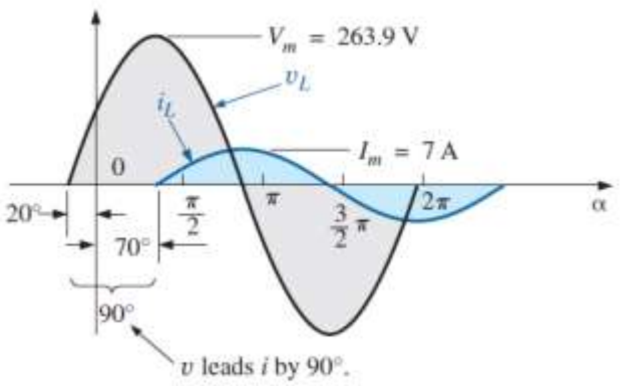
$$\upsilon = 263.9 \sin(377t - 70° + 90°)$$

$$\upsilon = 263.9 \sin(377t - 70° + 90°)$$

and

$$\boldsymbol{\upsilon = 263.9 \sin(377t + 20°)}$$

The curves are sketched below:



Example 4: The voltage across a 0.5 H coil is provided below. What is the sinusoidal expression for the current?

$$v = 100 \sin 20t$$

Solution:

$$X_L = \omega L = (20 \text{ rad/s})(0.5 \text{ H}) = 10\ \Omega$$

$$I_m = \frac{V_m}{X_L} = \frac{100 \text{ V}}{10\ \Omega} = 10 \text{ A}$$

and we know the i lags v by 90°. Therefore,

$$i = \mathbf{10 \sin(20}t \mathbf{- 90°)}$$

Example 5: The voltage across a 1 μF capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves.

$$v = 30 \sin 400t$$

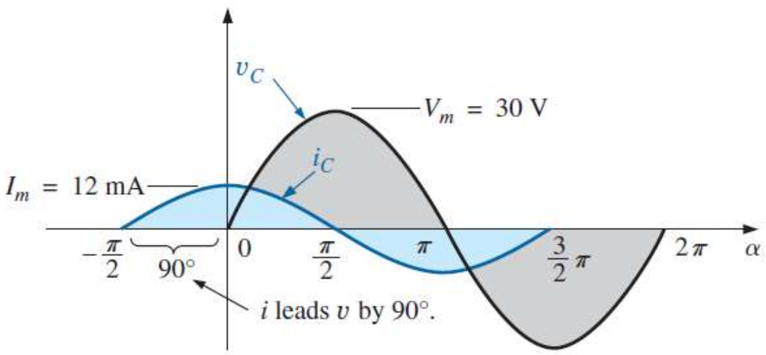
$$X_C = \frac{1}{\omega C} = \frac{1}{(400 \text{ rad/s})(1 \times 10^{-6} \text{ F})} = \frac{10^6\ \Omega}{400} = 2500\ \Omega$$

$$I_m = \frac{V_m}{X_C} = \frac{30 \text{ V}}{2500\ \Omega} = 0.0120 \text{ A} = 12 \text{ mA}$$

and we know that for a capacitor i leads v by 90°. Therefore,

$$i = \mathbf{12 \times 10^{-3} \sin(400}t \mathbf{+ 90°)}$$

The curves are sketched below:



Example 6: The current through a 100 μF capacitor is given. Find the sinusoidal expression for the voltage across the capacitor.

$$i = 40 \sin(500t + 60°)$$

Solution:

$$X_C = \frac{1}{\omega C} = \frac{1}{(500 \text{ rad/s})(100 \times 10^{-6} \text{ F})} = \frac{10^6 \,\Omega}{5 \times 10^4} = \frac{10^2 \,\Omega}{5} = 20 \,\Omega$$

$$V_M = I_M X_C = (40 \text{ A})(20 \,\Omega) = 800 \text{ V}$$

and we know that for a capacitor, v lags i by 90°. Therefore,

$$v = 800 \sin(500t + 60° - 90°)$$

and

$$v = \mathbf{800 \sin(500}t \mathbf{- 30°)}$$

Example 7

For the following pairs of voltages and currents, determine whether the element involved is a capacitor, an inductor, or a resistor. Determine the value of C, L, or R if sufficient data are provided:

a. $\upsilon = 100 \sin(\omega t + 40°)$
$i = 20 \sin(\omega t + 40°)$

b. $\upsilon = 1000 \sin(377t + 10°)$
$i = 5 \sin(377t - 80°)$

c. $\upsilon = 500 \sin(157t + 30°)$
$i = 1 \sin(157t + 120°)$

d. $\upsilon = 50 \cos(\omega t + 20°)$
$i = 5 \sin(\omega t + 110°)$

Solutions:

a. Since υ and i are *in phase,* the element is a *resistor,* and

$$R = \frac{V_m}{I_m} = \frac{100 \text{ V}}{20 \text{ A}} = \mathbf{5\ \Omega}$$

b. Since υ *leads* i by 90°, the element is an *inductor,* and

$$X_L = \frac{V_m}{I_m} = \frac{1000 \text{ V}}{5 \text{ A}} = 200\ \Omega$$

so that $X_L = \omega L = 200\ \Omega$ or

$$L = \frac{200\ \Omega}{\omega} = \frac{200\ \Omega}{377 \text{ rad/s}} = \mathbf{0.53\ H}$$

c. Since i *leads* υ by 90°, the element is a *capacitor,* and

$$X_C = \frac{V_m}{I_m} = \frac{500 \text{ V}}{1 \text{ A}} = 500\ \Omega$$

so that $X_C = \frac{1}{\omega C} = 500\ \Omega$ or

$$C = \frac{1}{\omega 500\ \Omega} = \frac{1}{(157 \text{ rad/s})(500\ \Omega)} = \mathbf{12.74\ \mu F}$$

d. $\upsilon = 50 \cos(\omega t + 20°) = 50 \sin(\omega t + 20° + 90°)$
$= 50 \sin(\omega t + 110°)$

Since υ and i are *in phase,* the element is a *resistor,* and

$$R = \frac{V_m}{I_m} = \frac{50 \text{ V}}{5 \text{ A}} = \mathbf{10\ \Omega}$$

7- FREQUENCY RESPONSE OF THE BASIC ELEMENTS

a- **Resistor *R*:** For an ideal resistor, you can assume that *frequency will have absolutely no effect on the impedance level*, as shown by the response in Fig.(4):

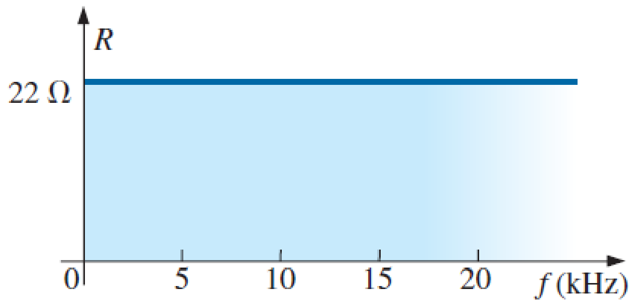


Fig.(4)

b- **Inductor *L*:** For the ideal inductor, the equation for the reactance can be written

$X_L = \omega L = 2\pi f L$

The response of X_L with frequency is shown in fig. (5)

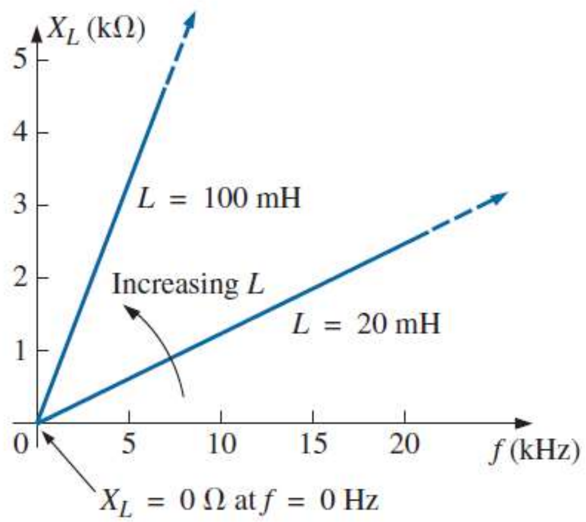


Fig.(5) X_L versus frequency.

at a frequency of 0 Hz, an inductor takes on the characteristics of a short circuit, as shown in fig.(6) by use the equation of X_L=2πfL:

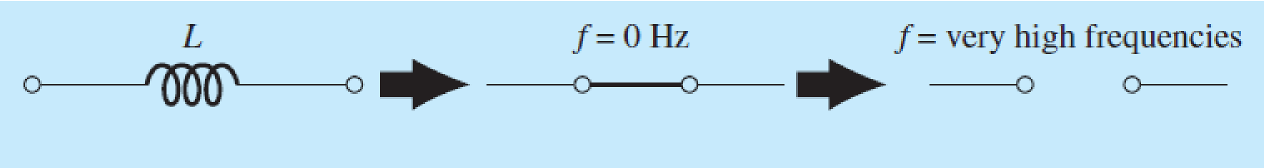


Fig.(6)

at very high frequencies, the characteristics of an inductor approach those of an open circuit, as shown in Fig.(6)

c- **Capacitor C**: For the capacitor, the equation for the reactance :

$$X_C = \frac{1}{2\pi f C}$$

The response of X_C with frequency is shown in fig. (7)

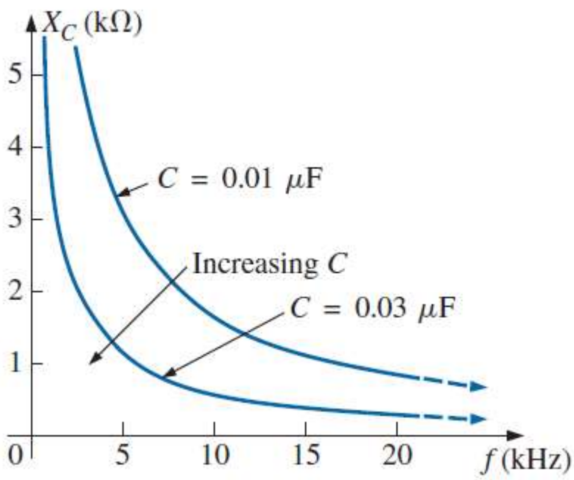


Fig.(7)

at or near 0 Hz, the characteristics of a capacitor approach those of an open circuit, as shown in Fig.(8)

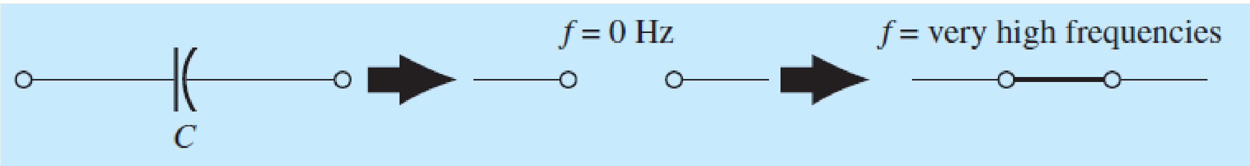


Fig.(8)

at very high frequencies, a capacitor takes on the characteristics of a short circuit, as shown in Fig.(8)

<u>Note</u>*: As frequency increases, the reactance of an inductive element increases while that of a capacitor decreases, with one approaching an open-circuit equivalent as the other approaches a short-circuit equivalent.*

<u>Example 8):</u> At what frequency will the reactance of a 200 mH inductor match the resistance level of a 5 k**Ω** resistor?

Solution: The resistance remains constant at 5 kΩ for the frequency range of the inductor. Therefore,

$$R = 5000\ \Omega = X_L = 2\pi f L = 2\pi L f$$
$$= 2\pi(200 \times 10^{-3}\ \text{H})f = 1.257f$$

and

$$f = \frac{5000\ \text{Hz}}{1.257} \cong \mathbf{3.98\ kHz}$$

<u>Example 9)</u> :At what frequency will an inductor of 5 mH have the same reactance as a capacitor of 0.1 μF?

Solution:

$$X_L = X_C$$
$$2\pi f L = \frac{1}{2\pi f C}$$
$$f^2 = \frac{1}{4\pi^2 LC}$$

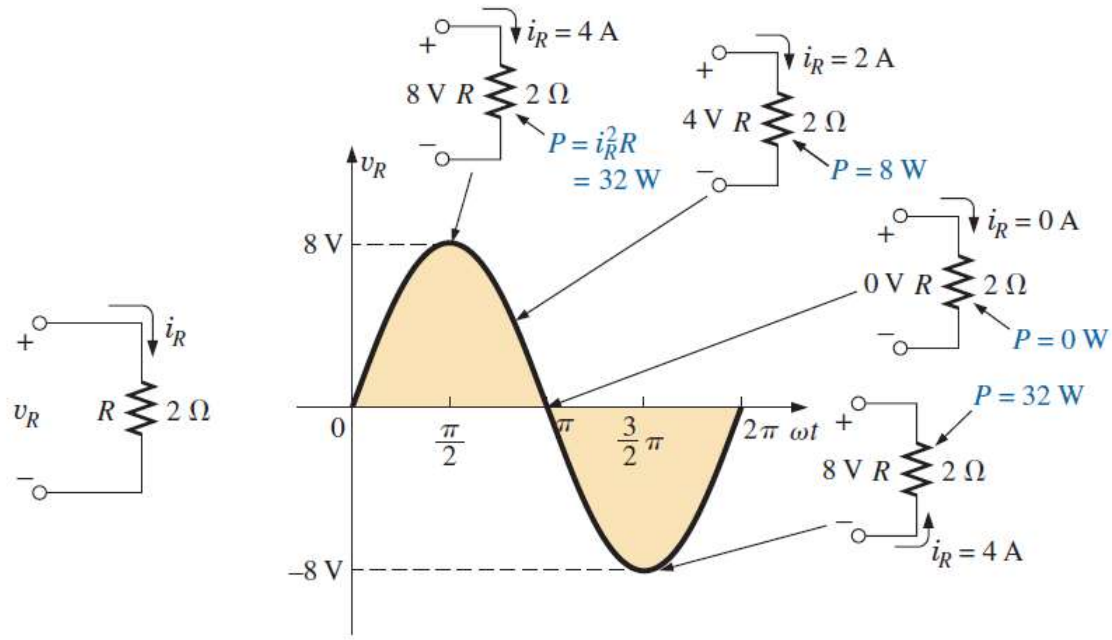
and

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5 \times 10^{-3}\ \text{H})(0.1 \times 10^{-6}\ \text{F})}}$$
$$= \frac{1}{2\pi\sqrt{5 \times 10^{-10}}} = \frac{1}{(2\pi)(2.236 \times 10^{-5})} = \frac{10^5\ \text{Hz}}{14.05} \cong \mathbf{7.12\ kHz}$$

8- AVERAGE POWER AND POWER FACTOR

For purely resistive load

consider the relatively simple configuration in Fig. (9) where an 8 V peak sinusoidal voltage is applied across a 2 **Ω** resistor. When the voltage is at its positive peak, the power delivered at that instant is 32 W as shown in the figure. At the midpoint of 4 V, the instantaneous power delivered drops to 8 W; when the voltage crosses the axis, it drops to 0 W. Note, however, that when the applied voltage is at its negative peak, the current may reverse but, at that instant, 32 W is still being delivered to the resistor.



Fig(9) *Demonstrating that power is delivered at every instant of a sinusoidal voltage waveform (except $V_R = 0$ V).*

1- ***Even though the current through and the voltage across reverse direction and polarity, respectively, power is delivered to the resistive load at each instant of time***

2- ***The fact that the power curve is always above the horizontal axis reveals that power is being delivered to the load at each instant of time of the applied sinusoidal voltage.fig.(10)***

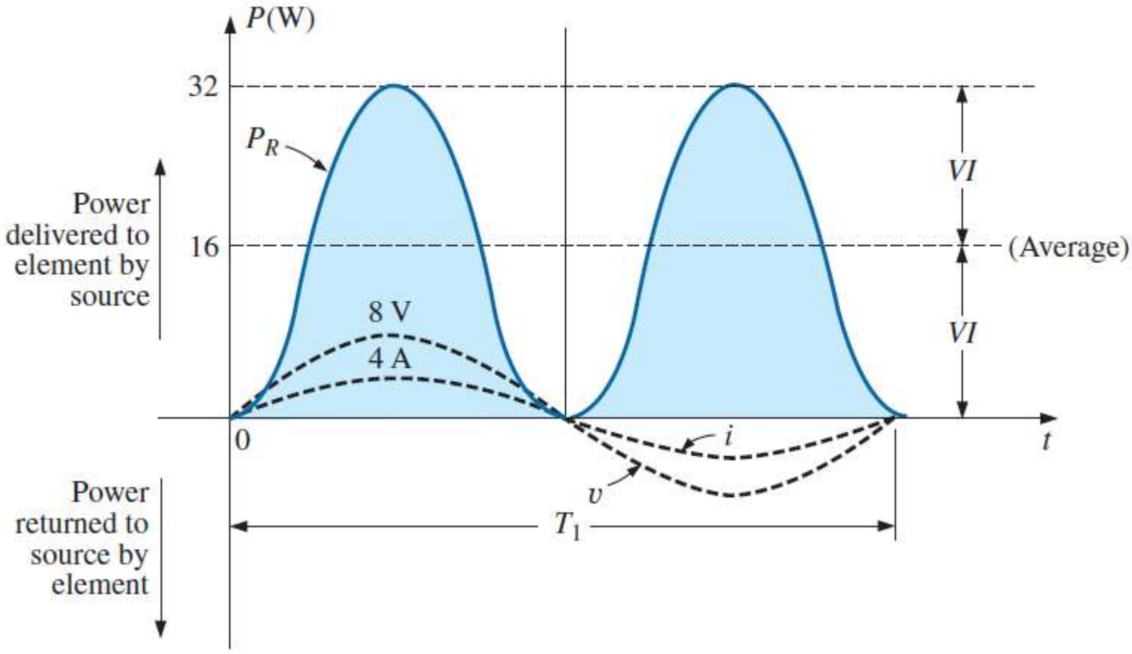


Fig.(10) *Power versus time for a purely resistive load.*

The average value of the power curve occurs at a level equal to(*Vm Im)*/2 as shown in Fig. (10) This power level is called the **average** or **real power** level. It establishes a particular level of power transfer for the full cycle.

$$P_{av} = \frac{V_m I_m}{2} = \frac{(\sqrt{2}\,V_{rms})(\sqrt{2}\,I_{rms})}{2} = \frac{2\,V_{rms} I_{rms}}{2}$$

$$P_{av} = V_{rms}\, I_{rms}$$

If the sinusoidal voltage is applied to a network with a combination of *R*, *L*, and *C* components, fig.(11):

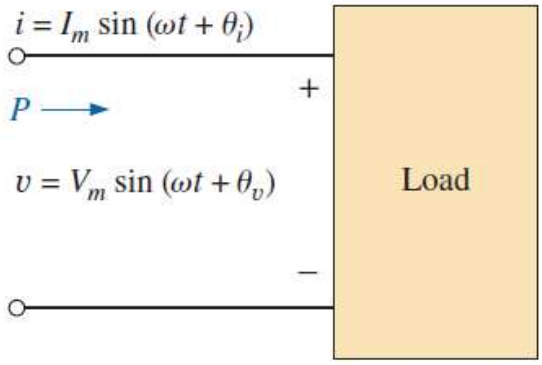


Fig.(11) *Determining the power delivered in a sinusoidal ac network.*

$$p = vi = V_m \sin(\omega t + \theta_v) I_m \sin(\omega t + \theta_i)$$
$$= V_m I_m \sin(\omega t + \theta_v) \sin(\omega t + \theta_i)$$

the magnitude of average power delivered is independent of whether v leads i or i leads v. fig.(12)

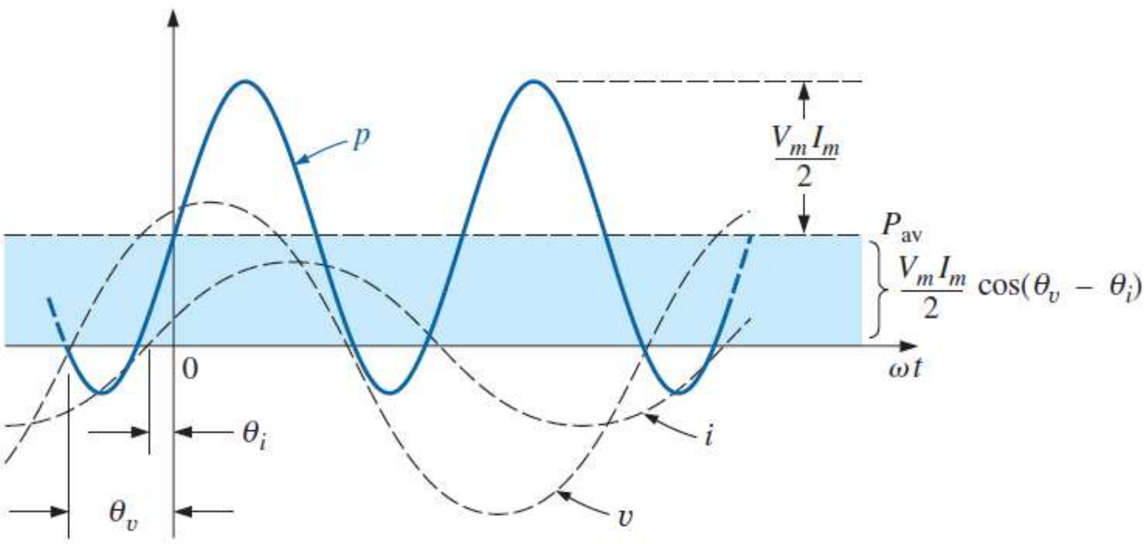


Fig.(12) *Defining the average power for a sinusoidal ac network.*

$$P = \frac{V_m I_m}{2} \cos\theta \qquad \text{(watts, W)}$$

where P is the average power in watts. This equation can also be written

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right) \cos\theta$$

or, since $\quad V_{\text{eff}} = \dfrac{V_m}{\sqrt{2}} \quad$ and $\quad I_{\text{eff}} = \dfrac{I_m}{\sqrt{2}}$

$$P = V_{\text{rms}} I_{\text{rms}} \cos\theta$$

Resistor

In a purely resistive circuit, since υ and i are in phase, $|\theta_\upsilon - \theta_i| = \theta = 0°$, and $\cos\theta = \cos 0° = 1$, so that

$$P = \frac{V_m I_m}{2} = V_{\text{rms}} I_{\text{rms}} \qquad \text{(W)}$$

Or, since $\quad I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R}$

then $\quad P = \dfrac{V_{\text{rms}}^2}{R} = I_{\text{rms}}^2 R \qquad \text{(W)}$

Inductor

In a purely inductive circuit, since υ leads i by 90°, $|\theta_\upsilon - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2} \cos 90° = \frac{V_m I_m}{2}(0) = \mathbf{0\ W}$$

The average power or power dissipated by the ideal inductor (no associated resistance) is zero watts.

Capacitor

In a purely capacitive circuit, since i leads v by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$. Therefore,

$$P = \frac{V_m I_m}{2}\cos(90°) = \frac{V_m I_m}{2}(0) = \mathbf{0\ W}$$

The average power or power dissipated by the ideal capacitor (no associated resistance) is zero watts.

Example(10) Find the average power dissipated in a network whose input current and voltage are the following:

$i = 5\sin(\omega t + 40°)$

$v = 10\sin(\omega t + 40°)$

Solution: Since v and i are in phase, the circuit appears to be purely resistive at the input terminals. Therefore,:

$$P = \frac{V_m I_m}{2} = \frac{(10\text{ V})(5\text{ A})}{2} = \mathbf{25\ W}$$

or $$R = \frac{V_m}{I_m} = \frac{10\text{ V}}{5\text{ A}} = 2\ \Omega$$

and $$P = \frac{V_{rms}^2}{R} = \frac{[(0.707)(10\text{ V})]^2}{2} = \mathbf{25\ W}$$

or $$P = I_{rms}^2 R = [(0.707)(5\text{ A})]^2(2) = \mathbf{25\ W}$$

For the following example, the circuit consists of a combination of resistances and reactances producing phase angles between the input current and voltage different from 0° or 90°.

Example (11): Determine the average power delivered to networks having the following input voltage and current:

a. $v = 100\sin(\omega t + 40°)$
   $i = 20\sin(\omega t + 70°)$

b. $v = 150\sin(\omega t - 70°)$
   $i = 3\sin(\omega t - 50°)$

Solutions:

a. $V_m = 100, \quad \theta_v = 40°$
$I_m = 20\text{ A}, \quad \theta_i = 70°$
$\theta = |\theta_v - \theta_i| = |40° - 70°| = |-30°| = 30°$

and

$$P = \frac{V_m I_m}{2}\cos\theta = \frac{(100\text{ V})(20\text{ A})}{2}\cos(30°) = (1000\text{ W})(0.866)$$
$$= \mathbf{866\ W}$$

b. $V_m = 150\text{ V}, \quad \theta_v = -70°$
$I_m = 3\text{ A}, \quad \theta_i = -50°$
$\theta = |\theta_v - \theta_i| = |-70° - (-50°)|$
$= |-70° + 50°| = |-20°| = 20°$

and

$$P = \frac{V_m I_m}{2}\cos\theta = \frac{(150\text{ V})(3\text{ A})}{2}\cos(20°) = (225\text{ W})(0.9397)$$
$$= \mathbf{211.43\ W}$$

9- Power Factor

$$\text{Power factor} = F_p = \cos\theta$$

$$F_p = \cos\theta = \frac{P}{V_{rms} I_{rms}}$$

The terms leading and lagging are often written in conjunction with the power factor. They are defined by the current through the load. If the current leads the voltage across a load, the load has a leading power factor .If the current lags the voltage across the load, the load has a lagging power factor.

Note: capacitive networks have leading power factors, and inductive networks have lagging power factors.

EXAMPLE(12)

Determine the power factors of the following loads ,and indicate whether they are leading or lagging .

a. Fig.1

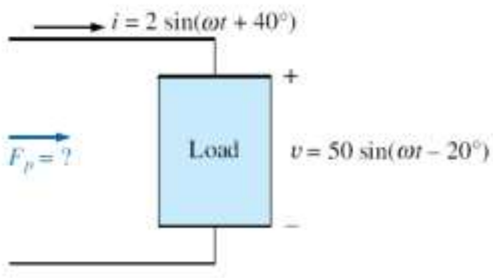


Fig.1

b. Fig.2

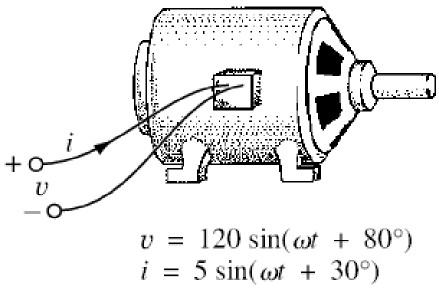


Fig.2

c. Fig.3

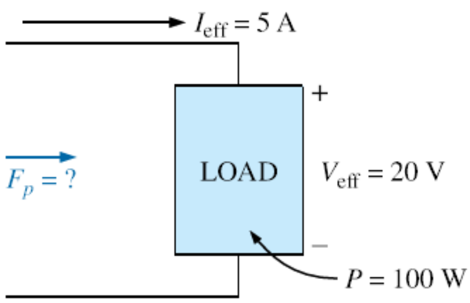


Fig.3

Solutions:

a. $F_p = \cos\theta = \cos\left|40° - (-20°)\right| = \cos 60° = \mathbf{0.5\ leading}$

b. $F_p = \cos\theta\ \left|80° - 30°\right| = \cos 50° = \mathbf{0.64\ lagging}$

c. $F_p = \cos\theta = \dfrac{P}{V_{\text{eff}}I_{\text{eff}}} = \dfrac{100\ \text{W}}{(20\ \text{V})(5\ \text{A})} = \dfrac{100\ \text{W}}{100\ \text{W}} = \mathbf{1}$

The load is resistive, and F_p is neither leading nor lagging.